Game Theoretic Models of Public Choice and Political Economy

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Contents

Declaration

I certify that this thesis does not incorporate any material previously submitted for a degree or diploma in any University; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person where due reference is not made in the text.

I also certify that the thesis has been composed by myself and that all the work is my own.

Paolo Balduzzi, September 2005

Abstract

This thesis is composed of three chapters, which can be read independently.

In the first one, we present and solve some bargaining games a la Rubinstein, where the subjects can delegate the negotiating process to agents. Delegation is aimed to provide the delegating party with a higher bargaining power. When both parties delegate, uncertainty arises about the final distribution of the payoffs and multiple equilibria are possible. The seller loses his usual first mover's advantage. When we allow for delegation costs, the range of multiple equilibria shrinks. The final outcome of the game may be now inefficient for the principals and a prisoners' dilemma may arise.

In the second chapter, we develop a model of simultaneous and sequential voting in a committee where members do not share their private information and do not have the same preferences. When objective functions differ, an optimal order of voting in the sequential game is found, leading to a unique socially optimal equilibrium. Our result rationalizes the presence of biased (i.e., partisan) voters in small committees as a way of reaching social optimality.

Finally, in the third chapter, we acknowledge that, beside the traditional public-private dichotomy for the provision of public services, an increasing attention has been devoted to the use of partnerships. We compare relative inefficiencies of public provision, traditional private provision and PPPs. We also analyze the effect of workers' efforts and incentives on the success of this device.

Chapter 1

Introduction

Playing games is a fascinating activity.

Children learn to use their hands, their body and their senses through ludic activities. They also develop social relations and improve their knowledge of the world. By growing up, we change our approach to games but not our willingness to play them. As adults, competition is probably the leading feeling and force behind our behaviour.

It is because we love games so much that we devoted an entire Ph.D. thesis to them. And it is because we like economics (but maybe not as much) that we ended up with a natural choice: like grown-up children, we use game theory to improve our knowledge of the world. And to learn how to use economic tools for succeeding in this activity.

Of course, our task in this job is not simply "playing", but setting up environments where hypothetical players can act and decide their strategies. We are not here to win anything: but our players are. We create games in different situations and we explore the best strategies available to our agents.

We also chose game theory for this particular reason: since its formalization, which we can date back to seminal contributions by John Von-Neumann, Oskar Morgenstern and John Nash, game theory has been used to study, explain and evaluate probably every possible matter, even beyond imagination. For instance, it has been applied to the study of evolution in biology or to the analysis of military strategies. It has been employed even to describe the relation between God and man[1]. For our purposes, game theory provides us with tools and foundations for explaining what we regard as one of the main political economy problems: agency decisions. We want to solve political problems and we develop our research in three very peculiar cases. Nonetheless, we positively think that our findings, results and insights can be applied to a wider range of situations. The problems under analysis are the dynamics of negotiations and its possible inefficiencies (chapter two), the ability of influencing a voting process (chapter three) and the choice of a provision mechanism for public goods (chapter four).

The thesis is therefore composed of three chapters, which are organized in two parts. In the first part ("The Race for Power"), we analyze the behaviour of players whose aim is to increase their power, which will be more precisely defined below in two different ways. The second part of the thesis ("The Use of Power") is composed of only one chapter. This chapter deals with the problem of a partially benevolent public institution, which is exogenously given some power and has to use its authority to implement some policy. In particular, its choice is about the best provision mechanism for goods.

These three chapters correspond to as many papers, which can be read independently from each other. Strategic game theory is the setting for our first two papers (chapters two and three), while the third one (chapter four) is developed in the incomplete contracts

[1]See, respectively, the contributions by John Maynard Smith (1982), Thomas Schelling (1960) and Steven J. Brams (2003).

framework. For this reason, it was not straightforward to write a single introduction to the thesis. We decided to start by focusing on some common elements of the papers, which are now exposed.

First of all, it is clear that we are interested in theoretical models rather then applied ones. This does not mean we do not want, or cannot, use them for predictions and policy suggestions. On the contrary, we believe theory has a leading role in developing new policy instruments, helping the understanding of everyday life events by simplifying the world to its essential elements. We are more interested in a qualitative analysis of reality rather than in quantitative enquiries. We build models which provide policy suggestions and reasonable results. Whether and how these models can be tested, and these results can be confirmed, are tasks that we leave to future research.

Secondly, as mentioned above, chapters two and three share the presence of competition about power.

In the first of these chapters ("Delegation Games with Full Commitment"), we define power as the ability of obtaining a profitable price (a high price for the seller and a low price for the buyer) in an alternating offers bargaining situation. "Price" can have itself a wider meaning than the mere economic one. It is easy, if not straightforward, to imagine a situation where price is an economic value. For instance, when two subjects are bargaining about some good (e.g., a house, a car, and so on). In this case, the idea behind strategic delegation is that hiring an agent to carry out the bargaining process, binding him to fully commit to the terms of the agreement, and signing a publicly known contract, dramatically reinforces the principal's side. Yet, the same mechanism applies even to less obvious contests. Let's think about international relations among nations: these are usually carried out by ministers and/or diplomats. These people do not really have the personal power of ratifying any agreement. What they do is act as delegates for their president or government. Beside other reasons (for instance, lack of time or better skills), this solution might be justified exactly as a way of gaining bargaining power. To explain the intuition behind this device, we refer to the first main contribution in this area, which is Thomas Schelling's *"The strategy of conflict"* (1960). His research particularly fits our example about delegation in politics, as it was developed during the dramatic and well known cold war contest.

In the second of these chapters ("Voting Games with Private Information and Heterogeneous Players"), power is defined as the ability to influence an election outcome. In the literature, voting strategies have initially been studied as very simple ones. The seminal contribution in this area is the *"Essai sur l'application de l'analyse a la probabilite des decisions rendues la pluralite des voix",* by the marquis de Condorcet (1785). The limit of the traditional approach is that voting is thought to be always sincere (or *naive*), that is, people simply vote for their favourite option. Sometimes, the aim to influence an election outcome may in fact bring people to vote strategically rather than sincerely. In our opinion, the best examples to understand strategic voting are provided by political elections with two main blocks and a small (but maybe regionally strong) third party. For instance, this is the case in Britain's general elections (with Labour, Conservative and Liberal Democratic Party) and Italy's ones (Polo delle liberta', Ulivo and Rifondazione Comunista). When the party someone is supporting has no reasonable possibility of winning, then an elector might strategically decide to vote for the least of the evils and opt for the second best option.

Thirdly, both chapter three and four were written having in mind public bodies: the

Italian Constitutional Court in the first case and British public authorities in the second. Both chapters deal with the contrast between private incentives (winning an election, maximizing profits) and social welfare. Social welfare is again defined following different criteria. In chapter three, social welfare is associated to the probability that a committee takes the correct decision for the society. In chapter four ("Models of Partnerships"), we introduce an explicit social welfare function, which is simply the sum of the utility and profit functions of the subjects in that economy.

Finally, welfare analysis is a common element to all the chapters: whether it is in a general equilibrium context or simply in a partial equilibrium one, we think it is essential to evaluate different policies and to provide suggestions. As explained above, in chapter three social welfare refers to the ability of a committee to take a decision which is best for society or, in other words, which corresponds to the true state of nature. Deviations from this behaviour are possible for two reasons: the presence of players with different preferences and the impossibility to fully aggregate information. In chapter four, every subject acts without considering the external effects of his decisions on other players. Inefficiency takes the form of distorted levels of investments and originates as an externality. In chapter two, the struggle to gain bargaining power through delegation may lead to excessive costs. The total gains of the parties (traditionally referred to as "the pie") may indeed be lower than without delegation.

We now briefly introduce and summarize the three chapters of the thesis. More accurate introductions, information and motivations will be found at the beginning of each chapter.

Delegation Games with Full Commitment

The first meaning of power, analyzed in the chapter "Delegation games with full commitment", is about the ability of using strategies to gain bargaining strength. Bargaining games have found lots of applications in economic theory. In particular, economic and political behaviour are often modelled as alternating offers situations, where two or more parties engage in a time consuming negotiation process. Moreover, in real life, we observe that most bargaining games are played by agents acting for other people. Principals often delegate the bargaining process to agents for different reasons (saving time, higher search ability, better skills, etc.). For instance, house owners delegate the search for tenants to estate agents, savers usually leave their money administration to professional traders, a premier delegates some of his duties to his ministers, many football players, singers, TV showmen hire agents. One of the main questions in negotiation models is about the source of bargaining power and, above all, about the devices to increase it. Schelling (*The strategy of conflict*, 1960) was probably the first author to emphasize the role of delegation as a strategic device. In particular, he dealt with delegation as a commitment tactic, through which a player could bind himself to a strategy and force the other to believe his threats. Subsequent studies have formalized the intuition that one-sided delegation is very effective. In chapter one, we present some models of bargaining with bilateral delegation, that is where both the principals can hire an agent. We look for the existence of equilibria and for possible inefficiencies arising when we introduce delegation costs (e.g.: training, time consumption *et cetera*).

Voting Games with Private Information and Heterogeneous Players

The second meaning of power, developed in the chapter "Voting Games with Private Information and Heterogeneous Players ", concerns the ability of influencing the outcome of a voting process when information is given only as a private signal and players vote sequentially. It is not difficult to find real life examples of sequential voting games. The most evident case is probably the presidential election mechanism in the USA: the candidates are locally chosen through primaries and each local primary is set up on a different date. An interesting example comes from Italy as well. When expressing on particular topics, courts are composed by three members and, according to the code of penal procedure (art. 527), votes are gathered starting from the opinions of the less experienced members (seniority rule). The chapter mainly focuses on two aspects of the problem. The first aspect is the behaviour of socially oriented agents who are not informed and therefore wish not to influence too much the outcome of the voting process. The second aspect is the rationale for the presence of strongly biased members (e.g. with a clear political preference) in committees whose aim is in fact social welfare.

This chapter has been to an extent inspired by the observation of a particular Italian institution, the "Corte Costituzionale" (Constitutional Court), whose members are appointed according to different criteria and therefore possibly have different preferences about the outcome of their decisions. We believe that the results we obtain hold for any decision-making body composed of a limited number of members, like boards of directors and so on.

Models of Partnerships

The debate about public and private provision of public goods and services has always been lively, both in the political and in the academic arena. The importance of contractual relationships is now even more emphasized by the increasing relevance assumed by partnerships between public (central or local) authorities and private firms. In this paper, we refer in particular to Public-Private Partnerships (henceforth, PPPs). The aim of this work is to present the main theoretical contributions to this debate, to discuss some experiences across UK, to introduce our models of partnerships and staff's contribution to their performance and, finally, to provide policy suggestions. Workers are assumed to care about the good they provide, in the sense that they are consumers at the same time. We presents models of partnerships where the relevant investments (in assets, innovations or job effort) are undertaken both by the provider and by the workers.

A remark

Against the apparent mainstream custom in Ph.D. theses, this one is not officially dedicated to anyone and does not contain any proper thanking section. In fact, I owe a great deal to a lot of people for this work. Some of them are explicitly referred to in the opening footnotes of every single chapter. As for the others, I offer them my love and friendship, something I anachronistically prefer to a few words written on a thesis they will never read or will never understand.

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Part I

The Race for Power

Chapter 2

Delegation Games with Full Commitment

2.1 Introduction

Bargaining games have found lots of applications in economic theory. In particular, economic and political behaviours are often modelled as alternating offers situations. Moreover, in real life, we observe that most bargaining games are played by agents acting for other people. Principals often delegate the bargaining process to agents for different reasons (saving time, higher search ability, better skills, etc.). For instance, house owners delegate the search for tenants to estate agents; savers usually leave their money administration to professional traders; a premier delegates some of his duties to his ministers; many football players, singers, and TV showmen hire agents.

One of the main questions in negotiation models is about the source of bargaining power and, above all, about the devices to increase it. Schelling (*The strategy of conflict*, 1960) was probably the first author to emphasize the role of delegation as a strategic device. In particular, he dealt with delegation as a commitment tactic, through which a player could bind himself to a strategy and force the other to believe his threats. Subsequent studies have formalized the intuition that one-sided delegation is very effective.

In this chapter we present some models of bargaining with bilateral delegation. The aim of this work is to present, define, solve and comment on different delegation games among the following four subjects: a seller, a buyer, the seller's intermediary, and finally the buyer's intermediary. Our model is developed in the well-known alternating offers framework a la Rubinstein (1982). The structure of the game implies that the agents' profits depend upon both principals' proposals. The first consequence is that the initial Rubinstein game among principals becomes a typical Nash demand game. We show that, when delegation is costless, principals always delegate, either as a dominant strategy or as a Nash reply. Multiple equilibria arise and one of the two parties is always better off. This means that bilateral delegation can provide effective gains to at most only one of the two principals (the "winner" of the game). As regards the agents, they are always paid their reservation wage. Assuming that the seller has the usual first mover advantage, delegation is more likely to be profitable to the buyer. Indeed, he has less to lose by switching from the direct bargaining game to the delegation one.

These results are no longer true when we allow for costs, being them exogenous or endogenous. These costs reduce the range of possible equilibria arising. In some of these ranges, the delegation game can be characterized as a prisoner's dilemma, that is, the principals may decide to delegate even if they both lose with respect to a direct bargaining situation. The presence of costs also reduces the set of equilibria emerging. When delegation is costly for the agents, they are both paid at least their reservation wage. Nevertheless, I may have a strategic first mover advantage and he is able to gain more than J.

The chapter is organized as follows. After a review of the literature on this topic (section two), in section three we develop a baseline model of two-sided delegated bargaining without renegotiation. Then, we enrich the model by introducing exogenous delegation costs (section four) and endogenous opportunities (section five). The players' best strategies are shown to depend on the level of these costs and opportunities. Finally, in section six we discuss the limits of our model, suggest some possible developments and draw our conclusions[1].

2.2 Review of the literature

The study of bargaining has been developed through two main formal approaches: an axiomatic one and a strategic one. The former is based on the idea that a bargaining solution should satisfy some reasonable conditions (axioms). Its first development is due to Nash (1950; 1953). The latter relies on the fact that outcomes are the results of interdependent players' choices (Nash, 1951). The widest used framework in this case is due to Rubinstein (1982), who exploits the idea that bargaining is actually a process of alternating offers. Elements like the degree of impatience influence the final outcome of the game. Very good expositions of these models and of their limits are provided by Sutton (1986) and Osborne and Rubinstein (1990).

[1]I thank Jozsef Sakovics, John H. Moore and Jonathan Thomas for their suggestions and supervision. I also thank seminar participants at the SGP Peeble Conference 2003, in Edinburgh, in Milan-Bicocca and at the Scottish Economic Society Conference 2004 for their comments.

One of the main question in negotiation models is about the source of bargaining power and, above all, about the devices to increase it. Schelling (1956, 1960) is probably the first author to emphasize the role of delegation as a strategic device. In particular, he deals with delegation as a commitment tactic, through which a player can bind himself to a strategy and force the other to believe his threats. As he pointed out:

> *"if the buyer can accept an irrevocable commitment, in a way that is unambiguously visible to the seller, he can squeeze the range of indeterminacy down to the point most favorable to him"(Schelling, 1956; p. 283).*

The effectiveness of this tactic is clearly linked to its *irrevocability, credibility* and *transparency*: if a delegation contract can easily be renegotiated, if an intermediary can be convinced that he will be better off by accepting a different proposal, or if the characteristics of the delegation contract cannot be effectively communicated to the other party, then delegation turns out to be much less powerful than expected.

Subsequent research has formalized this simple but strong idea in different ways. As in the rest of the literature about bargaining, contributions follow either an axiomatic or a strategic approach. As explained in the introduction, our model is developed in an alternating offers framework.

The literature in this field is not very wide. Table 2.1 summarizes it and the rest of the section is devoted to its exposition. Our approach is to discriminate the different contributions according to the following characteristics: degree of the commitment, framework, number of agents, level of information and application to particular problems. A few of less relevant (for our purposes) papers are also illustrated at the end of this section.

Jones (1989) and Burtraw (1992) model two-sided delegation games in an axiomatic framework. These papers are both based on the idea that the way the subjects can increase their bargaining power is through misrepresentation of their preferences. The former develops a game where players (principals) bargain over the division of two private goods and only differ in their individual tastes. They both can hire agents to bargain on their behalf, and the choice of an agent corresponds to the choice of his individual taste parameters. Principals non-cooperatively choose the type of their agents. The agents' bargaining process is solved by a Nash solution. Two kinds of equilibria emerge. The first is a self representation one (all the subjects have the same individual taste parameter), where no gains from delegation arise. The second is given by the general case when principals differ in tastes. Conclusions about the welfare properties of this latter equilibrium are similar to ours: situations with inefficient outcomes arise (both parties lose; i.e., a prisoner's dilemma) along with situations where only one party is better off. As long as delegation is costless, inefficiency never emerges in our model.

Burtraw (1992) links the choice of an intermediary to risk aversion considerations, which is the most natural way to misrepresent one party's preferences. Delegating to a risk-neutral agent is undoubtedly effective (as well as, we can reasonably guess, delegating to less impatient agents in a Rubinstein framework). This idea was not actually new but had been developed only in one-sided delegation models. His contribution differs from Jones (1989) to the extent that the former explicitly relies on risk-aversion differences among players. As in our model, multiple equilibria arise. Unfortunately, the author is not interested in the welfare implications of his results.

Table 2.1: Review of the literature on bargaining and delegation

Topic	**Author**
1) Commitment and bargaining:	
a) Full commitment	Schelling (1956; 1960)
b) Partial commitment	Muthoo (1996)
2) Delegation:	
a) In an axiomatic framework	Jones (1989)
	Burtraw (1992)
b_1) In a strategic framework	Fershtman, Judd and Kalai (1991)
	Muthoo (1999)
	Polo and Tedeschi (2000)
b_2) In a Rubinstein framework	Bester and Sakovics (2001)
3) Delegation and renegotiation:	
a) Bilateral delegation	Polo and Tedeschi (2000)
	Muthoo (1999)
b) One-side delegation	Bester and Sakovics (2001)
4) Incomplete information	Katz (1991)
	Fershtman and Kalai (1997)
	Corts and Neher (2001)
5) Applications	Vickers (1985)
	Fershtman and Judd (1987)

Papers developing general models (two-sided delegation, general utility functions and compensation schemes) in a non-cooperative framework are provided by Fershtman, Judd and Kalai (1991), Polo and Tedeschi (2000) and Muthoo (1996; 1999).

Fershtman, Judd and Kalai (1991) show that strategic delegation introduces some co-operative elements in a typical non-cooperative game. They do not specify any particular form for the players' utility functions or compensation schemes (but they stress the importance of weak monotonicity for the latter) and show that principals may coordinate through their agents to obtain efficient outcomes in equilibrium. In our model, each agent gains at least his reservation price (or exactly his reservation price in the baseline model) as a result of individual (and not collusive) rationality constraints. Moreover, delegating may emerge as Nash equilibrium but never allows for a Pareto improvement in the principals' outcome.

Polo and Tedeschi (2000) generalize Fershtman, Judd and Kalai (1991) and consider non separable utilities. As a result, they obtain multiple equilibria, where all the individually rational allocations are in the set of equilibria (that is, not only the Pareto efficient ones). This multiplicity is ruled out by relaxing the assumption of full commitment (i.e., allowing for renegotiation). We obtain a multiplicity problem as well. The range of possible equilibria dramatically shrinks when we introduce costs.

Muthoo (1996) introduces costly revocable commitments, showing that higher revocation costs lead to a stronger bargaining position. In this model, he does not explain how commitment is reached. This drawback is overcome by Muthoo (1999), where the author extends his previous general model to delegation. Both principals can hire a negotiator, who is characterized by a positive number k. This number influences both the cost of revoking the partial commitment and the agent's wage. Principals choose k maximizing their own payoff functions. According to different shapes of the agents' wage function, we can observe either equilibria without delegation or equilibria with delegation. In the latter case, the unique Nash Equilibrium is Pareto inefficient. As far as this model allows for renegotiation, it is not directly comparable to ours.

These latest models analyze the delegation game as a strategic one. Nevertheless, they fail to explicitly consider the time-consuming process of negotiations (and possibly of renegotiation). Bester and Sakovics (2001) have developed a delegated bargaining game in an alternating offer framework. Our model is different to the extent that we allow for two-sided delegation (instead of one-sided) but not for renegotiation. A first result by Bester and Sakovics is that, when full commitment is possible (as in a Schelling-type situation), the delegating player (specifically, the seller) can gain the full pie, whereas his intermediary and the buyer cannot have anything. This conclusion is strictly based on the particular compensation scheme used, that is a fixed payment from I to S whenever trade occurs. In the same framework, we show that adding the possibility of two-sided delegation turns the initial situation into a typical Nash demand game. Their main result is that renegotiation does not fully wipe out the positive effects of delegation: this is still profitable when the cost of renegotiation is high and the cost of delegation is low. These costs are explicitly represented by the time which is consumed to reach an agreement between parties.

An interesting feature of delegating games is that, by allowing for the presence of additional players, inefficiency may arise. This inefficiency can be realized in two ways: lower total profits and possibility of disagreement. Only the first type of inefficiency is usually discussed. The possibility of disagreements, or stalemates, has been analyzed by

Crawford (1982) and it is based on the idea that irrevocability (of commitments) and uncertainty (of the principal's choice) matter. Even if the possibility of impasse in our model is not ruled out, this result never emerges in equilibrium.

Despite most of the contributions focus on generic models, strategic delegation has found natural application in oligopoly sequential games, as, for example, in Vickers (1985) and Fershtman and Judd (1987). The first shows that, in a typical "predation game", delegating to aggressive managers (maximizing market share) provides more profitable outcomes for the incumbent. A similar argument is discussed in Fershtman and Judd (1987): they show that, in oligopolies, the owner has an incentive to distort his management utility function from profit-maximization.

As already highlighted, delegation is meant to be effective as long as contracts are observable. Katz (1991) confirms this point of view, showing that most of the strategic advantages given by delegation may be lost with incomplete information. Corts and Neher (2001) challenge this result, pointing out that, despite incompleteness, Schelling's intuition is still correct with multilateral delegation and decentralized ownership. Also according to Fershtman and Kalai (1997) there are still benefits from delegation. These benefits depend on the type of delegation and the probability of observing the contract. In our model we assume complete information to keep the analysis as simple as possible.

Other contributions are not directly related to this chapter. Segendorff (1998a, 1998b) develops delegation games for the provision of a public good. Two populations (nations) delegate to agents (politicians) the negotiation process about the provision of a public and a private good. The direct bargaining situation (called autarchy) leads to an inefficiently small amount of the public good. This benchmark is compared with two delegation games, which differ according to the level (weak or strong) of authority given to the agents (i.e., the institutional set). A weak delegation game always provides a Pareto outcome whereas a strong one makes at least one of the two parties worse off. The latter result also emerges when delegation of power is determined endogenously. Rubinstein and Wolinsky (1987) deal with the presence of agents, but they do not present them as a strategic device of the principals. Gains in this model are obtained as the intermediary reduces the search costs, expressed in terms of time consumption. Finally, some experimental evidence about outcomes from delegation games is provided by Schotter, Zheng and Snyder (2000). In an attempt to explain some real world inconsistencies of face-to-face bargaining behaviour, as compared to laboratory experiments, they argue that a possible explanation may be the existence of intermediaries. By allowing for agents in their experiments, they can show how inefficiency arises (and how it is worsened by introducing for renegotiation) in the form of a longer time to find an agreement and higher opportunity costs.

2.3 A simple model of delegated bargaining

In this section, we develop a baseline case, where the subjects bargain about the sale of an indivisible good. Both principals can hire an intermediary: I for the seller S and J for the buyer B. An agent is hired by a principal with a contract: I should buy the good from S and sell it to the buyer; J should buy the good from the seller and then sell it to B. The valuation of the good is 0 to S, I and J and 1 to B. Every player has the same discount rate δ. Finally, we assume there is no possibility of renegotiation between a party and his intermediary, that is, delegation acts as full commitment. Bester and Sakovics (2001) have already shown that, when only one party can fully commit himself (specifically, S), he can gain the entire pie by signing a contract with I such that S will be paid a fixed amount f whenever trade will occur between I and B[2].

When both S and B can fully commit themselves to a strategy by hiring an intermediary, we expect delegation to be less effective. We are interested in understanding when, why and how, bilateral delegation is sustainable as equilibrium strategy.

In this first version of the model, delegation is completely costless. To make the exposition as clear as possible, we distinguish three stages:

(1) *The delegation stage*: in the first stage, the principals decide whether to delegate or not. If a principal decides to delegate, he makes a take-it-or-leave-it (TIOLI) offer to his own agent about the level of a fixed compensation scheme (f_S or f_B, with $f_n \in [0,1]$, $n = B, S$)[3]. If he decides to not delegate, he does not hire any agent. These decisions (hiring an agent and level of offers) are not made public until the bargaining stage begins.

(2) *The agents' stage*: each agent decides whether to accept or not his principal's offer on the basis of his own reservation wage and outside options. We assume in this section that there are no outside options and that, once an agent makes his decision (accept or reject), he is then committed to it[4]. As delegation is not costly, an agent's rejection and a principal's choice of not delegating have the same effect on the final outcome.

[2] It is easy to show that the same is true when only B can delegate: he will get the entire pie, whereas S and I will gain nothing (see the Appendix).

[3] In lemma 1, we demonstrate the optimality of fixed compensation schemes.

[4] This means that if an agent realizes *ex-post* that the compensation scheme he accepted would provide him a negative payoff, in the bargaining stage no acceptable offer will be made and no agreement will be reached. This perpetual disagreement (impasse) will imply no gains for all the players.

(3) *The bargaining stage:* in this stage, all the information is made public (specifically, presence of intermediaries, levels of f_S and f_B). The parties bargain over the price of the good according to an alternating offers model a la Rubinstein. A priori, there are three possible bargaining situations: bilateral delegation (when I bargains with J), one-sided delegation (I with B or S with J), and direct bargaining (S with B). The selling party (S or I) always makes the first offer.

Compensation schemes and profit functions in case of bilateral delegation are set as follows:

- I pays f_S to S whenever trade occurs and gains p_m;
- S gains f_S whenever trade occurs;
- B pays f_B to J whenever trade occurs and gains the value of the good (which is equal to 1);
- J pays p_m to I and gains f_B whenever trade occurs,

where f_S and f_B are the fixed compensation schemes decided by the principals in the delegation stage and are not renegotiable, and p_m (with $p_m \in [0, 1]$, $m = I, J$) is the selling price of the good (where m is the identity of the proposer of the price).

The agents' incentive to find an agreement in the bargaining stage is embodied by the assumption that their payoffs depend on the actual realization of the trade.

The extensive form of this game is presented in Figure 2.1. As illustrated above, the players' strategies give rise to four possible cases: I) bilateral delegation (D, D) when both S and B delegate *and* both the agents accept; II) and III) one-sided delegation (D, ND) and

(ND, D) when only one principal delegates *or* only one agent accepts; IV) direct bargaining (ND, ND) when both the principals do not delegate *or* both the agents reject the offer[5].

Figure 2.1: The delegation game

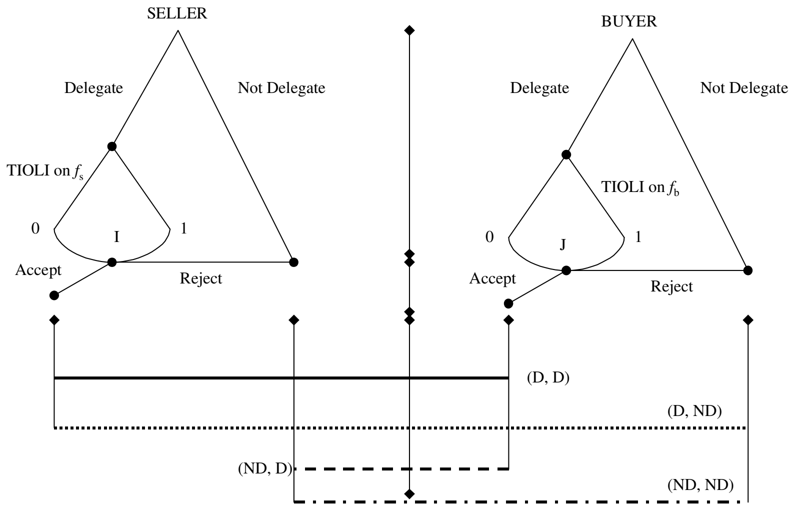


Table 2.2 summarizes the principals' payoff matrix at the initial delegation stage:

Table 2.2: Payoffs Matrix (I)

Seller - Buyer	D	ND
D	$f_S; 1 - f_B$	$f_S; \frac{\delta}{1+\delta}(1 - f_S)$
ND	$\frac{f_B}{1+\delta}; 1 - f_B$	$\frac{1}{1+\delta}; \frac{\delta}{1+\delta}$

When both the principals play ND, they gain the usual payoffs from direct bargaining. We call this the *default* situation. When only one party delegates, the payoffs can be obtained following Bester and Sakovics (2001)[6]. The payoffs from bilateral delegation can be worked out by the subsequent stages. Before taking any decision, the principals must therefore look at the consequences of two-sided delegation in stages (2) and (3).

[5]For sake of completeness, (ND, ND) can be a case also when only one principal decides to delegate but his offer is rejected.

[6]See Appendix.

The bargaining stage. The agents' choice in stage (2) is made on the basis of their possible gains in stage (3). If they reject, they gain their reservation wage, that is 0; if just one of them accepts and the other rejects, the latter gain the reservation wage and the former the gains according to Bester and Sakovics (2001). Finally, they can both accept. In this case, assuming stationary preferences, in each sub-period the bargaining parties (I and J) face the following situation:

$$\begin{cases} p_J - f_S = \delta\left(p_I - f_S\right) \\ -p_I + f_B = \delta\left(-p_J + f_B\right) \end{cases} \tag{2.1}$$

which means that, for each player, the gains from accepting the other party's offer today must equal the gains from rejecting it and make an offer tomorrow. In addition, since these gains must be non negative, the solutions of the system above are:

$$p_I = \frac{f_B + \delta f_S}{1+\delta} \quad p_J = \frac{\delta f_B + f_S}{1+\delta}$$

As I moves first and as the equilibrium is reached in the first period, p_I is accepted by J (i.e., p_I is the selling price). The payoffs for each player are: $\Pi_I = p_I - f_S = \dfrac{f_B - f_S}{1+\delta}$; $\Pi_J = f_B - p_I = \delta\dfrac{f_B - f_S}{1+\delta}$; $\Pi_S = f_S$; $\Pi_B = 1 - f_B$. The payoff matrix for all the players in the four possible situations of Figure 2.1 is given by Table 2.3.

As the players' payoffs are now clear, we introduce Lemma 1, which explains the rationality behind the use of fixed compensation schemes.

Lemma 1 *In bilateral delegation games, a fixed compensation scheme is profit maximizing for a principal and also the best response to the other principal's strategy.*

Proof. *A principal cannot extract the entire surplus by his compensation scheme as the other principal is delegating as well. Nevertheless, he can still maximize his profit by leaving the minimum to the agents. He can do this through a fixed payment compensation scheme. Since the difference* $(f_B - f_S)$ *constitutes the net gain the agents can share, both B and S wish to minimize it. For this reason, when a principal is playing this strategy, the other principal's best response is to ask for a fixed payment as well. As long as this difference is non negative, each agent has no incentive to reject any offer. Then, as the agent's payoffs (good or money) are conditional to an agreement, this is finally reached.* ■

Table 2.3: Payoffs Matrix (II)

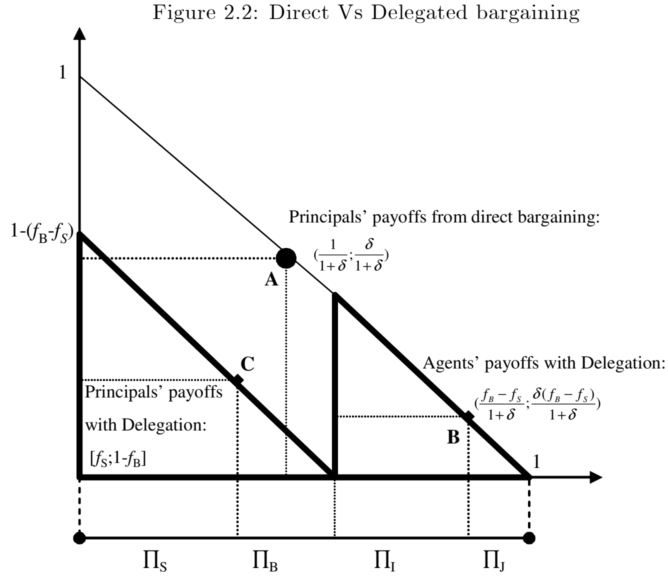
	Π_S	Π_B	Π_I	Π_J
(D, D)	f_S	$1 - f_B$	$\frac{f_B - f_S}{1+\delta}$	$\delta \frac{f_B - f_S}{1+\delta}$
(D, ND)	f_S	$\frac{\delta}{1+\delta}(1 - f_S)$	$\frac{1-f_S}{1+\delta}$	0
(ND, D)	$\frac{f_B}{1+\delta}$	$1 - f_B$	0	$\frac{\delta}{1+\delta} f_B$
(ND, ND)	$\frac{1}{1+\delta}$	$\frac{\delta}{1+\delta}$	0	0

The agents' stage. Consider the choice faced by the agents. Take for instance agent I. If he rejects his principal's proposal about f_S, he gains 0; if he accepts, he may gain $\frac{(f_B - f_S)}{1+\delta}$ or $\frac{(1-f_S)}{1+\delta}$ if he bargains with J or B respectively. So, he will accept any compensation f_S providing him with a non negative profit. For bilateral delegation to be a possible equilibrium, the following rationality (participation) constraints, for I and J respectively, must be simultaneously satisfied:

$$\begin{cases} \dfrac{f_B - f_S}{1+\delta} \geq 0 \\ \delta \dfrac{f_B - f_S}{1+\delta} \geq 0 \end{cases} \tag{2.2}$$

The difference $(f_B - f_S)$ cannot be negative: in the bargaining stage, the agents will realize they will gain negative payoffs in case of agreement. No compatible offers will be made and no agreement will be struck, leading all the players to the impasse point. Figure 2.2 compares two situations: bargaining without delegation (direct bargaining) and bargaining with delegation. We measure $S's$ and $I's$ payoffs on the horizontal axis and $B's$ and $J's$ payoffs on the vertical axis. A is the equilibrium outcome of a typical Rubinstein game between principals (direct bargaining): the position of this point only depends upon δ. In the second situation (equilibria B and C), two games are played: a Rubinstein one between agents, sharing a pie of $(f_B - f_S)$, and a Nash demand game between principals. B is the solution of the former game whereas C is the solution of the latter one. For sake of comparison, we project all the players' payoffs on a single horizontal segment

Figure 2.2: Direct Vs Delegated bargaining



There is obviously a conflict between S and B, as S wants to maximize f_S and B wants to minimize f_B.

Lemma 2 *Equilibria with bilateral delegation are supported only when* $f_B = f_S = f^*$*. This strategy gives the following profits:* $\Pi_I = 0$; $\Pi_J = 0$; $\Pi_S = f^*$ *and* $\Pi_B = 1 - f^*$.

Proof. *From (2.2) we know that* $f_B - f_S \geq 0$. *Suppose* $f_B > f_S$. *Then either the buyer would have an incentive to reduce* f_B *or the seller to increase* f_S. *Therefore, in equilibrium equality must hold. Finally, if* $f_B = f_S = f^*$, $\Pi_I = \Pi_J = 0, \Pi_S = f^*$ *and* $\Pi_B = 1 - f^*$ *from Table 2.3.* ■

As long as the agents do not have any outside option, they are not able to extract any extra-profit from the delegation contract. This result is actually not striking, as all the bargaining power was given to the principals.

We can now eventually focus on the principals' delegation stage.

The delegation stage. This is illustrated in Table 2.4, which updates and expands Table 2.2:

Table 2.4: Payoffs Matrix (III)

			Buyer					
			D					ND
			0	f_1	f_2	...	1	
Seller	D	0	$0;1$	$0;1-f_1$	$0;1-f_2$		$0;0$	$0;\frac{\delta}{1+\delta}$
		f_1	$0;0$	$f_1;1-f_1$	$f_1;1-f_2$		$f_1;0$	$f_1;(1-f_1)\frac{\delta}{1+\delta}$
		f_2	$0;0$	$0;0$	$f_2;1-f_2$		$f_2;0$	$f_2;(1-f_2)\frac{\delta}{1+\delta}$
		...						
		1	$0;0$	$0;0$	$0;0$		$1;0$	$1;0$
	ND		$0;1$	$\frac{f_1}{1+\delta};1-f_1$	$\frac{f_2}{1+\delta};1-f_2$		$\frac{1}{1+\delta};0$	$\frac{1}{1+\delta};\frac{\delta}{1+\delta}$

When a player decides to delegate, he sets a generic value f such that $0 \leq f \leq 1$. For simplicity, in Table 2.4 we consider only two intermediate values of f (with $f_1 < f_2$). This payoff matrix is set according to known results: payoffs for (ND, ND), for instance, are simply given by the usual Rubinstein model of direct bargaining, whereas gains for (ND, D) and (D, ND) are the known ones (see Appendix) and are valuated for the particular f. Our own model provides the gains for (D, D) and for the impasse points ($f_S > f_B$). The choice between delegating and not delegating constitutes a simple game between the two principals and the final outcome will depend on the level of f. The principals' best responses are as follows:

- If S plays D with a non negative f_S, then B always plays D setting $f_B = f_S = f^*$.
- If S plays D with $f_S = 1$, then B either plays D setting $f_B = f_S = 1$ or plays ND.
- If S plays ND then B always plays D setting $f_B = 0$.
- If B plays D with a strictly positive f_B, then S always plays D setting $f_S = f_B = f^*$.
- If B plays D with a $f_B = 0$, then S either plays D setting $f_S = f_B = 0$ or plays ND.
- If B plays ND then S always plays D setting $f_S = 1$.

Three cases emerge[7]. They are presented in Proposition 1, which also summarizes the results of this first section.

Proposition 1 *A two-sided delegation game is characterized as a typical Nash demand game. For extreme values of the compensation schemes, one-sided delegation equilibria are*

[7] For simplicity, we decide to focus exclusively on equilibria in pure strategies.

also supported. In particular, for each $f \in [0,1]$ *, there's a NEPS (Nash Equilibria in Pure Strategy) with bilateral delegation* (D, D)*; for* $f = 0$ *there is also a NEPS with one-sided delegation* (ND, D)*; finally, for* $f = 1$ *there is also a NEPS with one-sided delegation* (D, ND)*.*

Proof. Proposition 1 directly follows from the discussion above and from the solution of the game in Table 2.4. ■

Some discussion about these results is worthy. First of all, the presence of additional players (the agents) does not affect the size of the pie, that is, the total available profit. Of course, this result is strongly dependent on the no costs assumption.

Then, with regard to distributional issues, we first define the winner of the game.

Definition 1 (The winner of the game) *A principal is the winner of the delegation game when his payoff from delegating is bigger than his payoff from direct bargaining.*

When a principal expects the other principal to obtain the entire pie ($f = 0$ or $f = 1$), he has no strategic reason to look for an agent, as his payoff is going to be exactly the same. According to Definition 1, the buyer is winning from bilateral delegation as long as $f^* < \frac{1}{1+\delta}$; on the contrary, the seller is winning for higher values of f^*. There are more possible equilibria where the buyer is the winner for any $0 < \delta < 1$. That is, delegation fully destroys any first mover advantage for the seller. Indeed, this advantage is typical of time consuming games whereas the delegation one is characterized as a Nash demand game.

In the next sections, we will discuss the consequences of relaxing the important assumption that delegation is costless. First, we consider exogenous costly delegation for the players. Then, we analyze the effects of endogenous costly delegation. We expect the range of bilateral delegation equilibria to shrink or even disappear.

2.4 Exogenous costly delegation

We want to understand the robustness of this multiplicity result. In particular, we analyze the effects of introducing into the model some costs for the players. First, we assume costs to be exogenous and we distinguish between principals' and agents' costs.

2.4.1 Costly delegation for principals

Let us suppose that, simply because a principal employs an intermediary, he suffers a strictly positive fixed cost c independently from the fact that trade occurs or not. In particular, c is paid by the principal only if he decides to delegate. For instance, c may be the time cost of looking for an intermediary.

For simplicity, we assume $c_S = c_B = c$ to be the principals' common cost of delegation. How will the equilibria of the previous section be affected? First of all, we update the payment scheme in case of bilateral delegation:

- I pays f_S to S whenever trade occurs and gains p_m;
- S pays c when delegating and gains f_S whenever trade occurs;
- B pays c when delegating and f_B to J whenever trade occurs and gains the value of the good (which is equal to 1);
- J pays p_m to I and gains f_B whenever trade occurs

The range of equilibria in the previous section is modified as stated in Proposition 2.

Proposition 2 *In a two-sided delegation game with exogenous delegation costs c for the principals, the following equilibria emerge:* (D,D), (D,ND) *and* (D,ND) *for* $0 \leq c \leq \frac{\delta}{(1+\delta)^2}$; (D,ND) *and* (D,ND) *for* $\frac{\delta}{(1+\delta)^2} < c \leq \frac{\delta}{1+\delta}$; (ND,D) *for* $\frac{\delta}{1+\delta} < c \leq \frac{1}{1+\delta}$; *and* (ND,ND) *for* $c > \frac{1}{1+\delta}$. *The presence of costs is a source of inefficiency. In particular, in some intervals bilateral delegation equilibria are characterized as prisoner's dilemmas.*

Proof. The rest of this subsection is devoted to proving these results. ■

We notice that nothing changes for I and J. The level of c does not affect their reservation wage or strategy and the rationality constraints observed in (2.2) apply unchanged. Therefore we should only update the payoffs matrix in Table 2.4 and analyze the new delegation stage between B and S. The difference between Table 2.4 and Table 2.5 is that all the payoffs from delegating are reduced by the cost c.

Again, with bilateral delegation the condition $f_S = f_B = f^*$ must be satisfied.

Table 2.5: Payoffs Matrix (IV)

			Buyer				
			D				ND
			0	f_1	...	1	
Seller	D	0	$-c; 1-c$	$-c; 1-f_1-c$		$-c; -c$	$-c; \frac{\delta}{1+\delta}$
		f_1	$-c; -c$	$f_1-c; 1-f_1-c$		$f_1-c; -c$	$f_1-c; (1-f_1)\frac{\delta}{1+\delta}$
		...					
		1	$-c; -c$	$-c; -c$		$1-c; -c$	$1-c; 0$
	ND		$0; 1-c$	$\frac{f_1}{1+\delta}; 1-f_1-c$		$\frac{1}{1+\delta}; -c$	$\frac{1}{1+\delta}; \frac{\delta}{1+\delta}$

The players' best responses are now functions of c:

- If S plays D then $\begin{cases} \text{B plays } D \text{ if he expects} & f_S < 1 - c(1+\delta) \\ \text{B plays } ND \text{ if he expects} & f_S > 1 - c(1+\delta) \end{cases}$

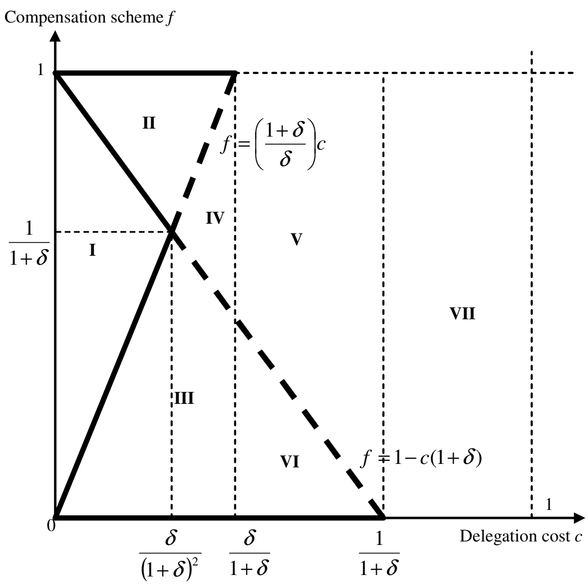
- If S plays ND then $\begin{cases} \text{B plays } D \text{ setting } f = 0 \text{ if} & c < \frac{1}{1+\delta} \\ \text{B plays } ND \text{ if} & c > \frac{1}{1+\delta} \end{cases}$

- If B plays D then $\begin{cases} \text{S plays } D \text{ if he expects} & f_B > c\left(\frac{1+\delta}{\delta}\right) \\ \text{S plays } ND \text{ if he expects} & f_B < c\left(\frac{1+\delta}{\delta}\right) \end{cases}$

- If B plays ND then $\begin{cases} \text{S plays } D \text{ setting } f = 1 \text{ if} & c < \frac{\delta}{1+\delta} \\ \text{S plays } ND \text{ if} & c > \frac{\delta}{1+\delta} \end{cases}$

We can still apply the same logic we used to solve the game in Table 2.4. There are 7 possible combinations of f and c arising. The simplest way to study and analyze them is to look at Graph 2.1, where we express the compensation scheme f as a function of the delegation cost c.

Graph 2.1: Equilibria with principals' costs



We start focusing on intervals where (D, D) is a solution. From the best response functions above, it emerges that any f^* in case (I) can be sustained as an equilibrium compensation scheme. In other words, bilateral delegation is sustained whenever $c\left(\frac{1+\delta}{\delta}\right) < f^* < 1 - c(1+\delta)$. Of course, this interval is shrinking as c grows and eventually collapses to 0 when $c = \frac{\delta}{(1+\delta)^2}$. At this point, $f^* = \frac{1}{1+\delta}$. For higher values of c, we have $1 - c(1+\delta) < c\left(\frac{1+\delta}{\delta}\right)$ and therefore (D, D) is no longer a possible equilibrium. On the contrary, (ND, ND) is the unique solution for any $c > \frac{1}{1+\delta}$: delegation is too costly for the principals and even one-sided delegation equilibria are ruled out.

In cases (I), (II), (III), and (VI) one-sided delegation equilibria arise as well. In these equilibria, the delegating party obviously plays his profit maximizing level of f, that is $f_S = 1$ or $f_B = 0$. All the uncertainty disappears once c is big enough to rule out multiplicity: for $\frac{\delta}{1+\delta} < c < \frac{1}{1+\delta}$, ND is a dominant strategy for the seller, no matter the expected value of f_B. This situation is obviously exploited by the buyer, who plays his profit maximizing $f_B = 0$. Finally, when $c > \frac{1}{1+\delta}$, both the players prefer ND.

Welfare implications: distributional and efficiency issues

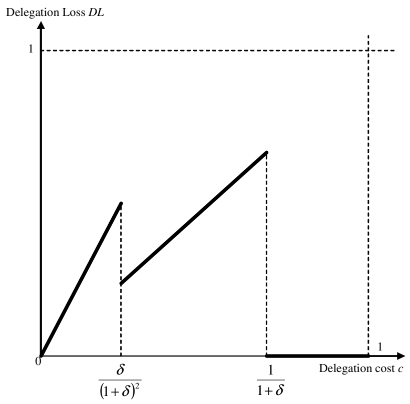
When it is costly, delegation is without doubt inefficient, as it shrinks the size of the pie available to the principals. Nevertheless, D is still played in some intervals.

To see the extent to which delegation is inefficient, we can define the delegation loss DL as the difference between the size of the non-delegating equilibrium total outcome (i.e., 1) and the delegating equilibria total outcome ($1 - c$ or $1 - 2c$ with one-sided and bilateral delegation respectively). This difference clearly depends on the level of the costs c. For simplicity and expositional purposes, we focus on the worst possible case and assume bilateral delegation equilibria are played when one-sided delegation equilibria are possible as well. When both the players delegate, a total loss of $2c$ is produced. For central values of the delegation costs, only one-sided delegation equilibria are sustainable, therefore the loss is equal to c. Finally, for larger c, neither principal delegates and there are no losses.

Graph 2.2 shows the non monotonic behaviour of the delegation loss function, which is defined by (2.3):

$$DL = \begin{cases} 2c & \text{for} \quad c < \frac{\delta}{(1+\delta)^2} \\ c & \text{for} \quad \frac{\delta}{(1+\delta)^2} < c < \frac{1}{1+\delta} \\ 0 & \text{for} \quad c > \frac{1}{1+\delta} \end{cases} \tag{2.3}$$

Graph 2.2: The delegation loss function



It is also interesting to understand who is the winner[8] of the delegation game. In the basic model, for instance, S is the winner whenever $f > \frac{1}{1+\delta}$. Now the discussion is a little bit more complex, as payoffs depend both on c and f. Graph 2.3 helps us to understand the situation.

The two thick straight lines are the functions $f = \frac{1}{1+\delta} - c$ and $f = \frac{1}{1+\delta} + c$. The little triangle $\overset{\triangle}{BCE}$ represents the gains from delegation for the seller[9]. The big triangle

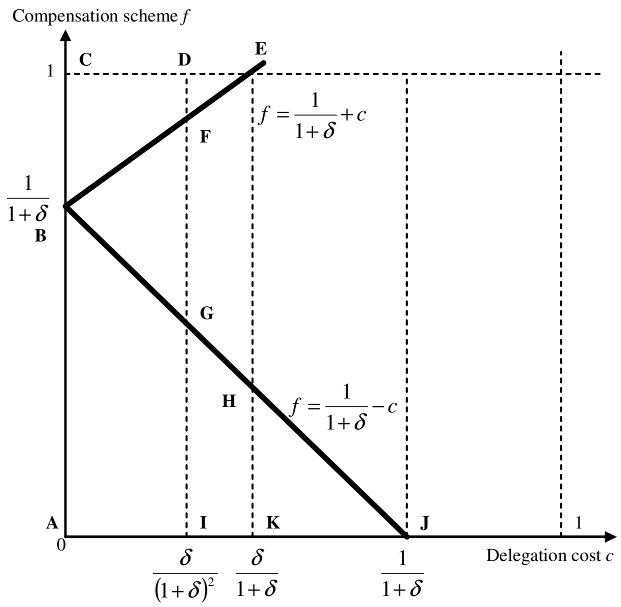
[8] See Definition 1.

[9] We simply call gains from delegation the difference between the player's payoff when he delegates and

$\overset{\triangle}{ABJ}$ gives the profits from delegation for the buyer. We start by focusing on the interval $0 < c < \frac{\delta}{(1+\delta)^2}$ and (D, D) equilibria. By Definition 1, S is the winner whenever $f - c > \frac{1}{1+\delta}$, that is whenever $f > \frac{1}{1+\delta} + c$ (area $BCDF$): the payoffs from delegating are higher than the payoffs from direct bargaining. Similarly, B is the winner if $1 - f - c > \frac{\delta}{1+\delta}$, that is, $f < \frac{1}{1+\delta} - c$ (area $ABGI$). In the triangle $\overset{\triangle}{BFG}$, that is when $\frac{1}{1+\delta} - c < f < \frac{1}{1+\delta} + c$ both the players are losing from delegation. This means that for some values of c and f, the delegation game is similar to a prisoner's dilemma: both the players would be better off by not delegating but strategic behaviours force them towards a Pareto dominated equilibrium.

For completeness, we should note that the game played along the vertical axis is simply the case we studied in the previous section, that is, costless delegation ($c = 0$). As c grows, the "both losing" space enlarges.

Graph 2.3: The winner of the game (I)



his payoff from direct bargaining.

When $\frac{\delta}{(1+\delta)^2} < c < \frac{\delta}{1+\delta}$, only one-sided delegation equilibria are possible: if (D, ND) is played, we are in $\overset{\triangle}{DEF}$; if (D, ND) is played, we are in $GIKH$. As these equilibria are *ex ante* equally possible, the delegation game is similar to a chicken (or hawk-dove) game. Then, for $\frac{\delta}{1+\delta} < c < \frac{1}{1+\delta}$, only (D, ND) is played and we are in the little triangle $\overset{\triangle}{HKJ}$: the buyer is the only possible winner. The existence of this area is quite surprising. For some values of c, the party that is usually the weakest (that is, the second mover) is stronger. We recall that we had a similar kind of asymmetry for $c = 0$. We can dare to give a new interpretation to Schelling's intuition that "weakness is often strength". This is not true only when a player can bind himself to a strategy. It also matches with the following (common sense) statement: weakness is strength when the starting payoff is lower (i.e., the player has less to lose).

Finally, for higher values of c, nobody finds delegation profitable.

2.4.2 Costly delegation for agents

Delegation may be costly for agents, too. We call their common cost y. This is known *ex ante* by all the players and it is suffered only once the agent accepts the delegation contract. We may think of y as the job effort. Or it can be the time spent to arrange meetings with the counterpart. I and J will accept a compensation scheme only if it will provide them with at least this value y. This cost is not sunk until the agent accepts a contract. In fact, if it were sunk before, a principal could leave all the burden upon the agent and the problem would be the same as in the basic model.

The range of equilibria shrinks as y grows. For high values of the agents' costs, delegation never occurs. A detailed characterization of the possible equilibria and corresponding relevant intervals of y is provided by Proposition 3. The rest of the subsection is then devoted to its explanation.

Proposition 3 *In a two-sided delegation game with symmetric exogenous delegation costs y for the agents, the following equilibria emerge: (D,D), (D,ND) and (ND,D) for $0 \leq y \leq \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$; (D,ND) and (ND,D) for $\frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)} < y \leq \frac{\delta}{(1+\delta)^2}$; and (ND,ND) for $y > \frac{\delta}{(1+\delta)^2}$. For some intervals of y, delegation is played even if it provides the principals with a lower payoff than in the direct bargaining case. Finally, agent I obtains a first mover advantage.*

Proof. The rest of the subsection is devoted to proving these results. ■

When delegation is costly, there is still room for agreements between principals and intermediaries. This is determined by the level of these costs. The agents can force the principals to give up some of their gains to hire them. Agent I is also able to gain more than his own cost y.

Existence of equilibria with two-sided delegation

When both the principals delegate, the agents' rationality constraints are now given by (2.4), which updates (2.2):

$$\begin{cases} \dfrac{f_B - f_S}{1+\delta} \geq y \\ \delta \dfrac{f_B - f_S}{1+\delta} \geq y \end{cases} \tag{2.4}$$

Both conditions must be satisfied simultaneously[10]. B and S cannot set the difference $(f_B - f_S)$ equal to 0 any longer; the most they can do is set it such that they satisfy the

[10] Suppose the difference $(f_B - f_S)$ is set such that J will not be able to gain any positive profit in the bargaining stage. Then, as already argued above, this choice will lead the game to the impasse point, where everybody has a zero payoff.

stricter of the constraints:

$$f_B - f_S = \left(\frac{1+\delta}{\delta}\right) y \geq (1+\delta)\, y \quad \forall \delta \in [0,1] \tag{2.5}$$

Lemma 3 *In any bilateral delegation equilibrium, the players' payoffs will be:* $\Pi_I = \dfrac{y}{\delta}$; $\Pi_J = y$; $\Pi_S = f_S$ *and* $\Pi_B = 1 - f_B$.

Proof. *The logic of Lemma 2 still applies.* ■

From the agents' point of view, there is an important difference. Agent I can gain more than the level y of his costs. This is a consequence of I being the first mover in the bargaining stage with J. As both the constraints in (2.4) must be satisfied, $(f_B - f_S)$ must be set so that it satisfies the stricter of the constraints, which is the second mover's constraint.

The principals play the following game: if S delegates he gains f_S; if he does not delegate but B delegates he gets $\frac{f_B}{1+\delta}$. If B delegates he gains $1 - f_B$; if he does not delegate but S delegates he gets $\frac{\delta}{1+\delta}\,(1 - f_S)$.

Taking into account (2.5), bilateral delegation equilibria will be sustained whenever:

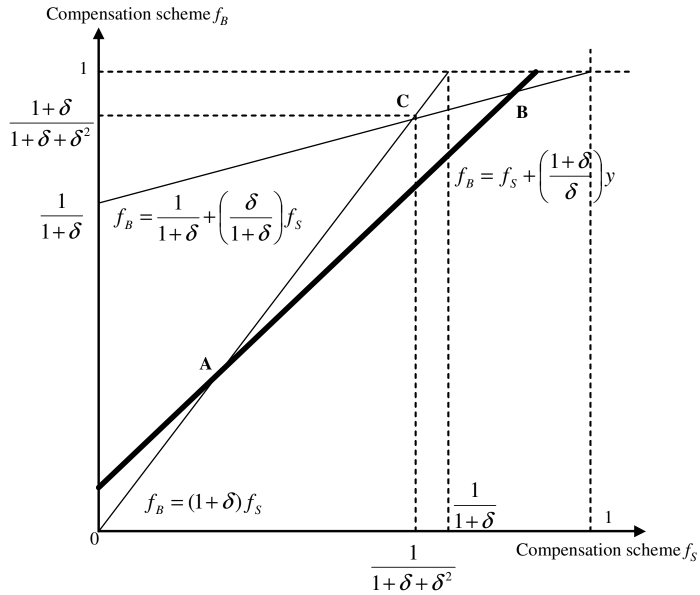
$$\begin{cases} f_S > \dfrac{f_B}{1+\delta} \\ 1 - f_B > \dfrac{\delta}{1+\delta}\,(1 - f_S) \\ f_B - f_S = \left(\dfrac{1+\delta}{\delta}\right) y \end{cases} \text{that is} \begin{cases} f_B < f_S\,(1+\delta) \\ f_B < \dfrac{1}{1+\delta} + \dfrac{\delta}{1+\delta} f_S \\ f_B = f_S + \left(\dfrac{1+\delta}{\delta}\right) y \end{cases} \tag{2.6}$$

The two inequalities in (2.6) respectively represent the rationality constraints for the seller and the buyer to play bilateral delegation. The last function (equality) is the agents' participation constraint (2.5). We depict all these functions in Graph 2.4, in order to illustrate the range of possible bilateral equilibria in terms of compensation schemes.

The inequalities determine a possible area $\overset{\triangle}{ABC}$ where bilateral delegation is an equilibrium. As the agents' participation constraint is binding (see Lemma 3), the range of equilibria reduces to the segment $\overline{AB}$. The presence of y influences the position of the thick line $f_B = f_S + \left(\frac{1+\delta}{\delta}\right) y$. Indeed, when $y = 0$, we know that any $f_S = f_B = f^*$ can be sustained as equilibrium (and indeed the segment $\overline{AB}$ sustains every $f \in [0,1]$). For higher values of y, this set of equilibria shrinks (the thick straight line shifts upwards) and eventually collapses to a singleton (point C) when $y = \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$. In this case: $f_S = \Pi_S = \frac{1}{1+\delta+\delta^2}$ and $f_B = \frac{1+\delta}{1+\delta+\delta^2}$ ($\Pi_B = 1 - f_B = \frac{\delta^2}{1+\delta+\delta^2}$).

For intermediate values of y, any compensation scheme between A and B is sustainable as bilateral delegation equilibrium. In terms of $f_{S,}$ the range of equilibria is given by: $\left(\frac{1+\delta}{\delta^2}\right) y \leq f_S \leq 1 - \frac{(1+\delta)^2}{\delta} y$. In terms of $f_{B,}$ this corresponds to: $\left(\frac{1+\delta}{\delta}\right)^2 y \leq f_B \leq 1 - (1+\delta)\, y$

Graph 2.4: Equilibria with exogenous agents' costs



Welfare implications of bilateral delegation We wish to obtain some insights about the welfare implications of these equilibria. In Graph 2.5 we present the same information as in Graph 2.4, but with respect to the principals' profits[11]. Condition (2.6) becomes:

$$\begin{cases} 1-\Pi_B < \Pi_S(1+\delta) \\ 1-\Pi_B < \dfrac{1}{1+\delta}+\dfrac{\delta}{1+\delta}\Pi_S \\ 1-\Pi_B = \Pi_S + \left(\dfrac{1+\delta}{\delta}\right) y \end{cases} \text{ that is } \begin{cases} \Pi_B > 1-\Pi_S(1+\delta) \\ \Pi_B > \dfrac{\delta}{1+\delta}-\dfrac{\delta}{1+\delta}\Pi_S \\ \Pi_B = 1-\Pi_S - \left(\dfrac{1+\delta}{\delta}\right) y \end{cases} \tag{2.7}$$

The presence of a strictly positive cost for the agents is always inefficient from the principals' point of view. Indeed, both of them must give up a share of the entire pie when hiring an agent. Though, the distribution of payoffs is such that, for some values of y, at least one of the two parties is better off.

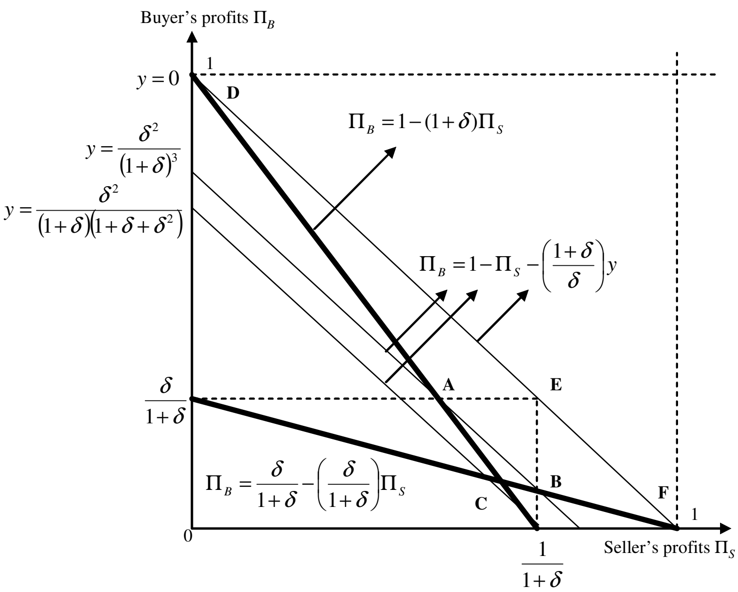
The thick straight lines in the graph represent the principals' rationality constraints to delegate. In this case, the space of bilateral delegation lies above the two lines. The thin straight lines represent the agents' participation constraints. In this case, higher values of y imply a downward shift of the line. In Graph 2.5 we depict three of these lines. The highest is for $y=0$. The middle one is for $y=\frac{\delta^2}{(1+\delta)^3}$. The lowest one is for $y=\frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$. Given Definition 1, we can state that for $0 \leq y \leq \frac{\delta^2}{(1+\delta)^3}$, the buyer is the winner whenever his payoff Π_B is bigger than $\frac{\delta}{1+\delta}$ (any compensation scheme on the segment $\overline{DE}$). On the contrary, the seller is the winner whenever $\Pi_S = f_S > \frac{1}{1+\delta}$ (segment $\overline{EF}$). Nevertheless, equilibria where a prisoner's dilemma arises are possible. In these equilibria $1-f_B < \frac{\delta}{1+\delta}$ and $f_S < \frac{1}{1+\delta}$. This interval is represented by any segment similar to $\overline{AB}$, that is, any portion of agents' participation constraint line in $\overset{\triangle}{ABE}$. For $\frac{\delta^2}{(1+\delta)^3} \leq y < \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$,

[11] We recall that $\Pi_S = f_S$ and $\Pi_B = 1-f_B$.

both the principals still delegate even if they are both worse off. Finally, for higher values of y, bilateral delegation never occurs.

Again, with symmetric intermediaries' costs, the delegation game is characterized (at least for some intervals of y) as a typical prisoner's dilemma: B and S would be both better off by not delegating. As in the previous subsection, these equilibria arise as delegation is a dominant strategy in the interval. The difference with the case of symmetric principals' costs is that now the loss is not a deadweight one, as the intermediaries are gaining and sharing it.

Graph 2.5: The winner of the game (II)



Existence of equilibria with one-sided delegation

For some values of f_B, f_S and y, equilibria with one-sided delegation are also possible. If we go back to (2.6), it is clear that, say, S will prefer ND whenever he expects B to play $f_B > f_S(1+\delta)$. Likewise, B will prefer ND whenever he expects S to play $f_S < \frac{(1+\delta)f_B - 1}{\delta}$. We have the following cases:

(a) Only the buyer delegates: for this three players case, from Table 2.3 we know[12] that the agent J's participation constraint is $\frac{\delta}{1+\delta} f_B \geq y$ (with equality in equilibrium) and the buyer's rationality constraint is $1 - f_B \geq \frac{\delta}{1+\delta}$. Therefore we have: $\Pi_S = \frac{f_B}{1+\delta} = \frac{y}{\delta}$; $\Pi_J = y$; $\Pi_B = 1 - f_B = 1 - \left(\frac{1+\delta}{\delta}\right) y$ if $y \leq \frac{\delta}{(1+\delta)^2}$ and $\Pi_S = \frac{1}{1+\delta}$; $\Pi_J = y$; $\Pi_B = \frac{\delta}{1+\delta}$ if $y > \frac{\delta}{(1+\delta)^2}$. Suppose S wants to deviate and play D. He would gain $\frac{f_B}{(1+\delta)}$ evaluated in $f_B = \left(\frac{1+\delta}{\delta}\right) y$, that is $\frac{y}{\delta}$. As in the baseline model, for extreme values of f_B, the seller is indifferent about whether to delegate or not.

(b) Only the seller delegates: it can be easily shown[13] that now the agent I's participation constraint is $\frac{1}{1+\delta}(1 - f_S) \geq y$ (with equality in equilibrium) and the seller's rationality constraint is $f_S \geq \frac{1}{1+\delta}$. The resulting payoffs are $\Pi_S = f_S = 1 - (1+\delta)\, y$; $\Pi_I = y$; $\Pi_B = \frac{\delta}{1+\delta}(1 - f_S) = \delta y$ if $y \leq \frac{\delta}{(1+\delta)^2}$ and $\Pi_S = \frac{1}{1+\delta}$; $\Pi_I = y$; $\Pi_B = \frac{\delta}{1+\delta}$ if $y > \frac{\delta}{(1+\delta)^2}$. Suppose B wants to deviate and to play D. He would gain $\frac{\delta}{(1+\delta)}(1 - f_S)$ evaluated in $f_S = 1 - (1+\delta)\, y$, that is δy. Again, for extreme values of f_S, the buyer is indifferent about whether to delegate or not.

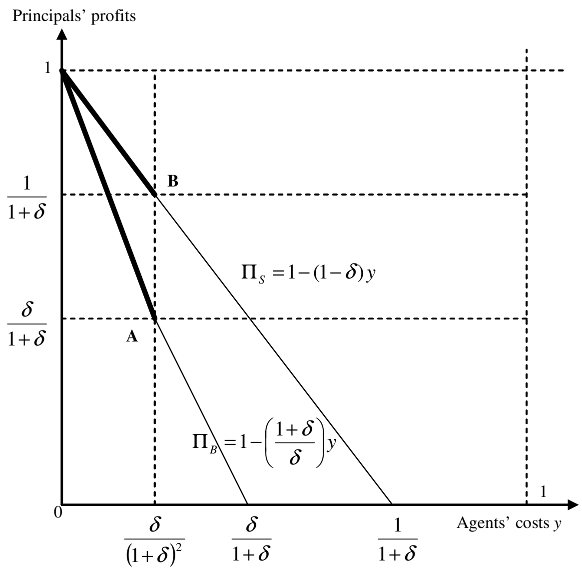
We recall from the previous subsection that gains from one-sided delegation equilibria were not evenly distributed between the principals. This is not true in this case: one-sided delegation equilibria are sustained in the same interval for both the principals. The previous asymmetry is now offset by an additional one. The weight of y is different whether this cost is sustained by the first mover or by the second one. The buyer has to give up more as he has to compensate a second mover agent with a strictly positive delegation cost.

[12] See Appendix.
[13] See Appendix.

Graph 2.6 shows how the level of y influences the principals' payoffs.

The upper downward sloping line represents the seller's payoffs when he is the only delegating principal. S will delegate as long as these are higher than his payoffs from direct bargaining, that is $\frac{1}{1+\delta}$. The lower downward sloping line represents the buyer's payoffs when he is the only delegating principal. B will delegate as long as his payoff from delegating is higher than his payoffs from direct bargaining, that is $\frac{\delta}{1+\delta}$. It is clearly shown by the graph that this happens for the same value of y, that is $\frac{\delta}{(1+\delta)^2}$.

Graph 2.6: One-sided delegation equilibria



For higher values of y, as already stated in Proposition 3, delegation never occurs.

2.4.3 A comparison between principals' and agents' costs

In the basic model without costs, we were concerned by the presence of multiple equilibria. These are still possible with positive levels of costs but their range is dramatically reduced even for very low levels of c and y. Nevertheless, the impact of c and y is different. In

Table 2.6 we compare the threshold values of these costs.

Agents' costs are more restrictive about the possibility of bilateral delegation equilibria. A first possible explanation is that, from the principals' point of view, a total agents' cost of $2y$ is actually paid as $y + \frac{y}{\delta} (> 2y)$ by the principals. Nevertheless, this is not fully satisfactory. For $\delta = 1$ the impact of the two costs is still different ($c = \frac{1}{4}$ and $y = \frac{1}{6}$ with bilateral delegation equilibria), even if the total cost is now exactly equal to $2y$. This first explanation only highlights the fact that the cost c is paid in a Nash demand game and, by construction, is independent of time. On the other hand, y is supported by individuals playing a Rubinstein game, where payoffs (and costs) are affected by elements like the degree of impatience (δ).

Table 2.6: Payoffs Matrix (V)

	Equilibrium		
	(D, D)	(D, ND)	(ND, D)
Max c	$\frac{\delta}{(1+\delta)^2}$	$\frac{\delta}{1+\delta}$	$\frac{1}{1+\delta}$
Max y	$\frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$	$\frac{\delta}{(1+\delta)^2}$	$\frac{\delta}{(1+\delta)^2}$

Another possibility is the following. From the principals' point of view, c is an *individual* cost. Suppose S has a positive c and B has not costs: only the seller's strategy will be affected. As regards y, it affects *both* the principals. Both of them have to support a share of this cost to be sure the agents will accept their offers. In addition, suppose I has no costs and J has a positive cost. As the only relevant constraint is the most binding one, the principals strategy would still be the same as in the case with symmetric agents' costs[14].

[14] In the opposite case, when only I has a positive delegation cost, the maximum y supporting bilateral delegation equilibria is $\frac{\delta}{(1+\delta)\left(1+\delta+\delta^2\right)}$ (which is still equal to 1/6 when $\delta = 1$).

2.4.4 A general model of exogenous costly delegation

In this subsection, we present a general model where delegation is costly both for the agents and the principals. We will only present the results and the main intuitions, as the way to solve the game has been already discussed in previous subsections.

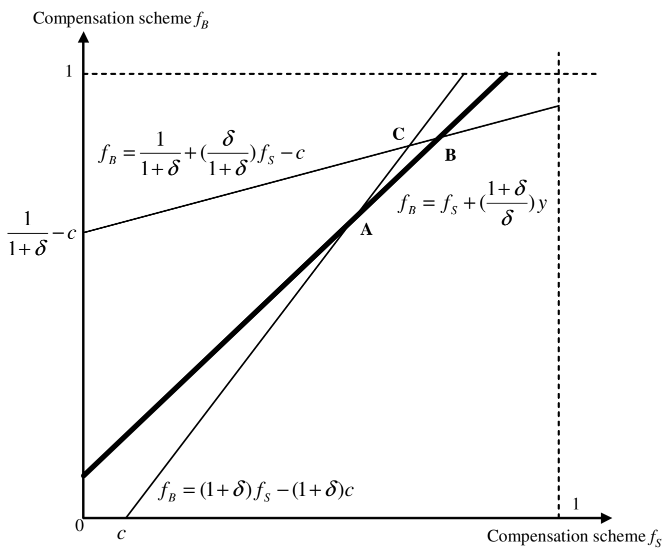
Existence of equilibria with bilateral delegation

The conditions for the existence of bilateral delegation equilibria are given by a modification of (2.6), where we subtract the delegation cost c from the principals' payoffs:

$$\begin{cases} f_S - c > \dfrac{f_B}{1+\delta} \\ 1 - f_B - c > \dfrac{\delta}{1+\delta}\left(1 - f_S\right) \\ f_B - f_S = \left(\frac{1+\delta}{\delta}\right) y \end{cases} \text{ that is } \begin{cases} f_B < \left(f_S - c\right)\left(1+\delta\right) \\ f_B < \dfrac{1}{1+\delta} - c + \dfrac{\delta}{1+\delta}\left(1 - f_S\right) \\ f_B = f_S + \left(\frac{1+\delta}{\delta}\right) y \end{cases} \tag{2.8}$$

Graph 2.7 shows how the range of multiple equilibria quickly shrinks.

Graph 2.7: Bilateral delegation equilibria with exogenous costs

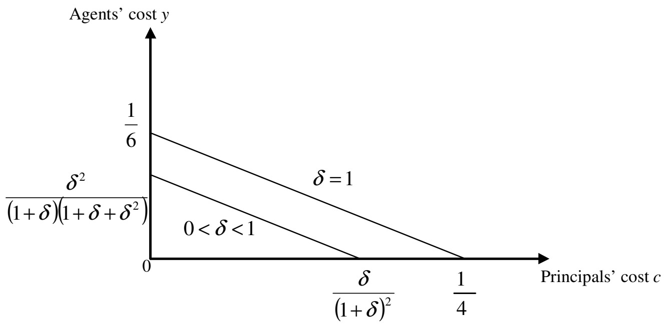


From a geometric point of view, the presence of c influences the position (intercept point with the axes) of the principals' constraints (thin lines). In particular, positive costs imply a downward shift of these lines. With regard to y, it moves the agents' constraint line upwards (thick line). From (2.8), we can work out a relation between the values of c and y such that bilateral delegation equilibria are possible. In particular, we have that:

$$y \leq \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)} - c\frac{\delta(1+\delta)}{1+\delta+\delta^2} \text{ or } c \leq \frac{\delta}{(1+\delta)^2} - y\frac{1+\delta+\delta^2}{\delta(1+\delta)} \tag{2.9}$$

This relation is graphically shown in Graph 2.8 for different values of δ. The higher the degree of impatience, the bigger the range of values of c and y which can support bilateral delegation equilibria. As we should expect from the previous subsection, for $c = 0$, the maximum level of exogenous costs supporting bilateral delegation equilibria is $y = \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$.

Graph 2.8: Costs, impatience and equilibria (I)



On the contrary, when $y = 0$, the maximum level of exogenous principal costs turns out to be $c = \frac{\delta}{(1+\delta)^2}$.

2.5 Delegation with endogenous opportunities

We now introduce a different kind of cost, that is an endogenous quantity x which can be consumed by the agents only once the delegation process is over. This additional pie of size x is shrinking according to the agent's own discount factor δ (we still assume that all the subjects have the same δ). That is, its actual size depends on the particular time the players reach an agreement. We will refer to x both as a cost, given that it influences the compensation scheme an agent will require, and as an opportunity, because it is a source of additional income. We can interpret x as a wage from a different job: an agent can decide to do just this job and gain x, or to accept the delegation contract and then go back to what remains of x.

Proposition 4 characterizes the set of equilibria of this game.

Proposition 4 *In a two-sided delegation game with endogenous delegation costs x for both agents, the following equilibria emerge: bilateral delegation equilibria for $0 \leq x \leq \frac{\delta^2}{(1+\delta)}$; (ND, D) equilibria for $0 < x \leq \frac{\delta}{1+\delta}$; (D, ND) equilibria for $0 < x \leq \frac{1}{1+\delta}$; and direct bargaining equilibria for $x > \frac{1}{1+\delta}$.*

Proof. The rest of the section will provide the proof for this Proposition. ■

2.5.1 Equilibria with bilateral delegation

When both the principals delegate, I and J will be engaged in the following process, which is a modification of the condition in (2.1):

$$\begin{cases} p_J - f_S + x = \delta \left(p_I - f_S + x \right) \\ f_B - p_I + x = \delta \left(f_B - p_J + x \right) \end{cases}$$

As usual, this system of simultaneous equations means that the gains from accepting the other party's offer today must equal the gains from rejecting it and making an offer tomorrow. The first equation refers to I and the second to J. If we consider I, for instance, his profits are determined by the selling price he gains from J, by the compensation scheme he must pay to S and by the opportunity x. As an agreement between I and J is struck in the first period, the relevant solution of the system above is given by I's first offer, that is:

$$p_I = \frac{f_B + \delta f_S}{1+\delta} + x\frac{1-\delta}{1+\delta}$$

The selling price p_I is now corrected by the presence of an additional term, that is $x\frac{1-\delta}{1+\delta}$. The presence of positive endogenous costs positively influences the equilibrium selling price. The way this influence works is clearer when we look at the payoffs for each of the players:

$$\begin{cases} \Pi_I = p_I - f_S + x = \dfrac{f_B - f_S + 2x}{1+\delta} \\ \Pi_J = f_B - p_I + x = \delta\Pi_I = \delta\dfrac{f_B - f_S + 2x}{1+\delta} \\ \Pi_S = f_S \\ \Pi_B = 1 - f_B \end{cases}$$

We can also express the agents' payoffs in a different way, highlighting the weight of this additional opportunity:

$$\begin{cases} \Pi_I = \dfrac{f_B - f_S}{1+\delta} + 2x\dfrac{1}{1+\delta} \\ \Pi_J = \delta\dfrac{f_B - f_S}{1+\delta} + 2x\dfrac{\delta}{1+\delta} \end{cases}$$

The total additional opportunity ($2x$) is split between the agents according to the usual

Rubinstein's rule. The agents do not simply bargain across the share of pie left by the principals $(f_B - f_S)$. They also consider their additional opportunity, which is external to the bargaining problem. In other words, an agent's payoff is influenced also by the other agent's opportunity. I's payoff is not only augmented by x; it is actually augmented by a quantity $2x\frac{1}{1+\delta} > x$. This means that I, as first mover, is also able to obtain some of J's endogenous opportunity. Looking back at the selling price p_I, in the bargaining process it seems that I gives up to a share $\frac{\delta}{1+\delta}$ of his x and gains a share $\frac{1}{1+\delta}$ of J's share (this can more easily be seen by allowing for different costs, say, x_I and x_J). Eventually, I has two bigger shares $(2x\frac{1}{1+\delta})$ and J two smaller ones $(2x\frac{\delta}{1+\delta})$.

We know that x plays the role of the agents' reservation wage as well; indeed, I and J may decide to refuse the delegation contract and to consume directly the quantity x. The following agents' rationality constraints must be simultaneously satisfied[15]:

$$\begin{cases} \dfrac{f_B - f_S + 2x}{1+\delta} \geq x \\ \delta\dfrac{f_B - f_S + 2x}{1+\delta} \geq x \end{cases}$$

that is,

$$f_B - f_S \geq x\frac{1-\delta}{\delta} \qquad \forall\delta \in [0,1] \tag{2.10}$$

The condition in (2.10) is different from the one in (2.5), when we introduced exogenous agents' costs. This is due to the fact that x is not only a positive reservation wage but also a source of additional profit within the bargaining process (i.e., it appears also on the left

[15] Again, if the difference $(f_B - f_S)$ is set such that J will not be able to gain any positive profit in the bargaining stage, then the game will move towards an impasse and everybody will get a zero payoff.

hand side of the inequality). In equilibrium, the principals fix the compensation schemes such that the equality holds:

Lemma 4 *In any bilateral delegation equilibrium, the players' payoffs will be:* $\Pi_I = \dfrac{x}{\delta}$; $\Pi_J = x$; $\Pi_S = f_S$ *and* $\Pi_B = 1 - f_B$. *The range of the principals' payoffs is as follows:* $\frac{1-\delta}{\delta^2}x \leq \Pi_S \leq 1 - \frac{(1-\delta)(1+\delta)}{\delta}x$; $(1-\delta)\,x \leq \Pi_B \leq 1 - \frac{(1-\delta)(1+\delta)}{\delta^2}x$.

Proof. The logic of Lemma 2 still applies to the first part of Lemma 4. As regards the second part, we just note that the game is solved through the system in (2.11):

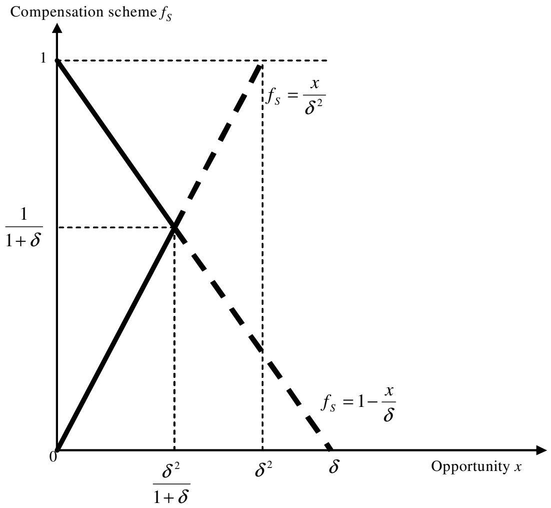
$$\begin{cases} f_S > \dfrac{1}{1+\delta} f_B + \dfrac{1}{1+\delta} x \\ 1 - f_B > \dfrac{\delta}{1+\delta}\left(1 - f_S + x\right) \\ f_B - f_S = x\dfrac{(1-\delta)}{\delta} \end{cases} \text{ that is } \begin{cases} f_B < f_S\left(1+\delta\right) - x \\ f_B < \dfrac{1}{1+\delta} - \dfrac{\delta}{1+\delta}x + \dfrac{\delta}{1+\delta} f_S \\ f_B = f_S + \left(\frac{1-\delta}{\delta}\right) x \end{cases} \tag{2.11}$$

The two inequalities determine the range of values of f_S and f_B supporting bilateral delegation equilibria. They require the principals' profits from bilateral delegation (left hand side) to be higher than profits from one-side delegation (right hand side). The particular nature of x is such that a principal can enjoy some of the other principal's agent opportunity even if the former is not delegating (right hand side of the inequalities). The final equality is the agents' participation constraint (which is binding) and directly comes from (2.10). We obtain equilibria with bilateral delegation for $x \leq \frac{\delta^2}{1+\delta}$ and $\frac{x}{\delta^2} \leq f_S \leq 1 - \frac{x}{\delta}$ (or, alternatively, $\frac{1+\delta-\delta^2}{\delta^2}x \leq f_B \leq 1 - x$). ■

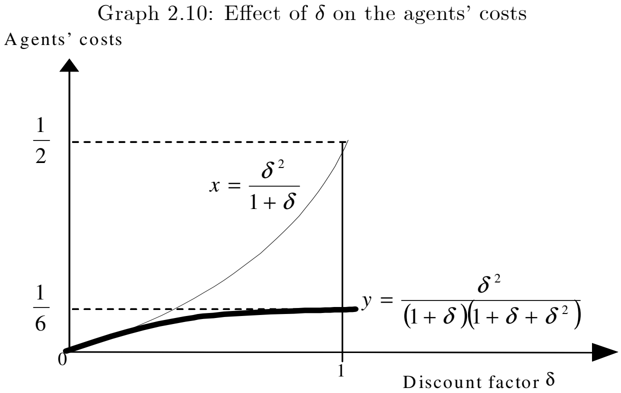
Graph 2.9 illustrates this interval in terms of x and f_S. The maximum value of x allowing bilateral delegation ($x = \frac{\delta^2}{1+\delta}$) is bigger than the value of the exogenous costs ($y = \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$) for any strictly positive δ.

Graph 2.10 compares these two different costs. In the relevant interval, that is for $\delta \in [0,1]$, y is concave while x is convex with respect to δ. When the subjects are very patient (δ close to 1), sensible levels of x can still support bilateral delegation equilibria. On the contrary, these are supported only for small values of y. The difference gets smaller the more the individuals become impatient.

Graph 2.9: Equilibria with endogenous opportunities



Graph 2.10: Effect of δ on the agents' costs



This impact is different as, with additional opportunities of profit (x), delegation is "less damaging" than with exogenous costs from the principals' point of view. In addition, for $x = \frac{\delta^2}{(1+\delta)}$, the payoffs of the principals are the following: $\Pi_S = \frac{1}{1+\delta}$ and $\Pi_B = \frac{1+\delta-\delta^2}{1+\delta}$, which are bigger (or equal) to their payoffs from direct bargaining (for any possible δ). The level of x positively influences the principals' payoffs and even for high values of these costs the principals are never worse off with respect to direct bargaining.

2.5.2 Equilibria with one-sided delegation

Equilibria with one-sided delegation are possible as well. We have the following cases:

(a) When only S delegates, B and I solve the following game:

$$\begin{cases} p_B - f_S + x = \delta\,(p_I - f_S + x) \\ -p_I + 1 = \delta\,(-p_B + 1) \end{cases}$$

The first equation refers to the usual seller's agent problem. The second equation explains the buyer's decision: he can accept the selling price proposed by I today and gain the value of the good, or reject it and propose a different selling price tomorrow. The agreement is reached in the first period, so the price which is paid is $p_I = \frac{1+\delta f_S}{1+\delta} - x\frac{\delta}{1+\delta}$; consequently:

$$\begin{cases} \Pi_I = p_I - f_S + x = \dfrac{1 + x - f_S}{1+\delta} \\ \Pi_B = -p_I + 1 = \delta \Pi_I \end{cases}$$

If S wants I to accept the job, he must design f_S such that I's profit from accepting the offer (left hand side of the inequality in (2.12)) is at least equal to the profit he obtains by refusing it:

$$\Pi_I = p_I - f_S + x \geq x \tag{2.12}$$

that is:

$$f_S \leq 1 - \delta x$$

In equilibrium the equality holds. As $\Pi_S = f_S$, then S will prefer delegation to non delegation as long as his payoffs from delegating are higher than his payoffs from not delegating:

$$1 - \delta x \geq \frac{1}{1+\delta}$$

that is:

$$x \leq \frac{1}{1+\delta}$$

So we have the following payoffs: $\Pi_S = f_S = 1 - \delta x$; $\Pi_I = x$ and $\Pi_B = \delta x$ if $x \leq \frac{1}{1+\delta}$; $\Pi_S = \frac{1}{1+\delta}$; $\Pi_J = x$; $\Pi_B = \frac{\delta}{1+\delta}$ if $x > \frac{1}{1+\delta}$. Is it profitable for B to play ND in this case? Is really (D, ND) an equilibrium? We must compare the following payoffs: δx, gained by B in the (putative) equilibrium (D, ND), and the payoff from deviating and playing D when S is delegating with $f_S = 1 - \delta x$. In the latter case, taking into account that the agents' rationality constraint in (2.10) must be satisfied, we work out $\Pi_B = 1 - f_B = \frac{\delta^2+\delta-1}{\delta}x$. By comparison, $\delta x \geq \frac{\delta^2+\delta-1}{\delta}x$ for any possible x and δ. Therefore (D, ND) is sustainable as equilibrium in the interval. There is also another case, that is what happens when S does not delegate because x is too high. This is equivalent to studying the case when only B delegates, which is shown in (b).

(b) We solve a similar three subjects game when only B delegates. We have: $\Pi_S = \frac{x}{\delta}$; $\Pi_J = x$ and $\Pi_B = 1 - f_B = 1 - \frac{x}{\delta}$ if $x \leq \frac{\delta}{1+\delta}$ and $\Pi_S = \frac{1}{1+\delta}$; $\Pi_I = x$; $\Pi_B = \frac{\delta}{1+\delta}$ if $x > \frac{\delta}{1+\delta}$. We apply the same logic as before and we compare $\frac{x}{\delta}$ (from the putative one-sided delegation equilibrium) and x (from S deviating to D). Again, $\frac{x}{\delta} \geq x$ for any possible x and δ. Therefore (ND, D) is sustainable as equilibrium in the interval.

One-sided delegation equilibria are therefore characterized as follows: (D, ND) and (ND, D) for $x \leq \frac{\delta}{1+\delta}$; (D, ND) for $\frac{\delta}{1+\delta} < x \leq \frac{1}{1+\delta}$; (ND, ND) for $x > \frac{1}{1+\delta}$.

The results of Proposition 4 directly follow.

The presence of agent's endogenous opportunities is actually a benefit for the principals: no prisoner's dilemma emerges. Actually, from the players' point of view, bilateral delegation equilibria weakly dominate direct bargaining equilibria, that is, every player is at least better off.

It is therefore very interesting to analyze a model with endogenous and exogenous agents' costs and understand which is the net effect.

2.5.3 A general model of costly delegation for agents

We present a general model where agents face both exogenous (y) and endogenous (x) costs. We will only present the results and the main intuitions, as the way to solve the game has been already discussed in previous subsections.

Existence of equilibria with bilateral delegation

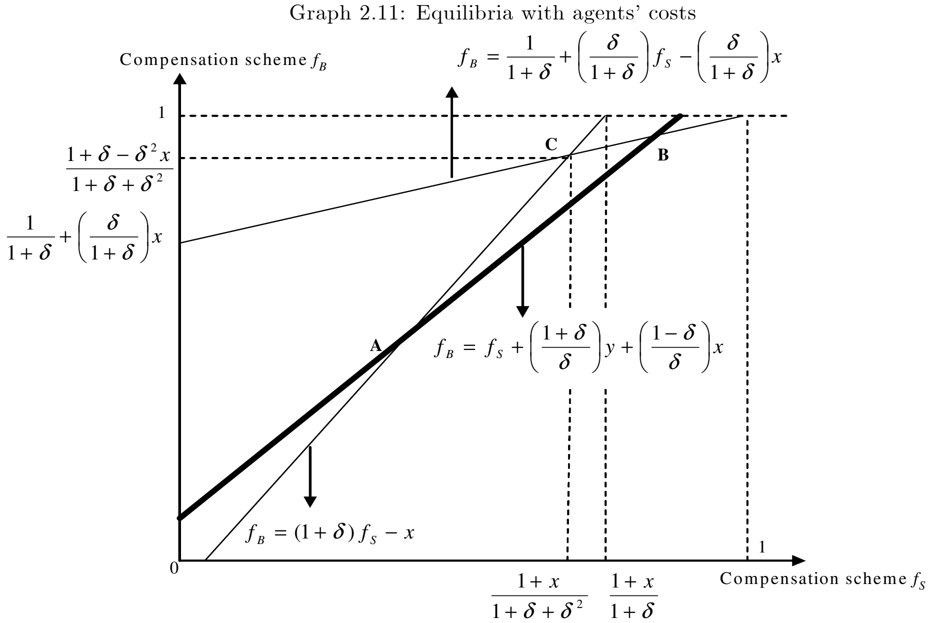
The conditions for the existence of bilateral delegation equilibria are given by:

$$\begin{cases} f_S > \dfrac{f_B + x}{1+\delta} \\ 1 - f_B > \dfrac{\delta}{1+\delta}\left(1 - f_S + x\right) \\ \delta \dfrac{f_B - f_S + 2x}{1+\delta} = x + y \end{cases} \text{ that is } \begin{cases} f_B < f_S\left(1+\delta\right) - x \\ f_B < \dfrac{1}{1+\delta} - \dfrac{\delta}{1+\delta} x + \dfrac{\delta}{1+\delta} f_S \\ f_B = f_S + \left(\frac{1-\delta}{\delta}\right) x + \left(\frac{1+\delta}{\delta}\right) y \end{cases} \qquad (2.13)$$

which updates (2.11). The only difference is in the agents' participation constraint (the equality). We consider the joint presence of y as positive reservation wage and of x as both a positive reservation wage and a source of additional profit within the bargaining process.

From a geometric point of view, the presence of y influences only the position of the agents' constraint line (the thick one in Graph 2.11) while the presence of x influences all the three conditions. The range of multiple equilibria shrinks the higher the values of x and y. The range of values of f_S and f_B sustaining bilateral delegation equilibria are found at the intersections between the agents' constraint and both of the principals' lines (the thick ones). These values are the following: $\frac{1}{\delta^2}x + \frac{1+\delta}{\delta^2}y \leq f_S \leq 1 - \frac{1}{\delta}x - \frac{(1+\delta)^2}{\delta}y$ and $\frac{1+\delta-\delta^2}{\delta^2}x + \frac{(1+\delta)^2}{\delta^2}y \leq f_B \leq 1 - x - (1+\delta)\,y$. They simply mix the range values we found with only endogenous or exogenous costs.

Graph 2.11: Equilibria with agents' costs

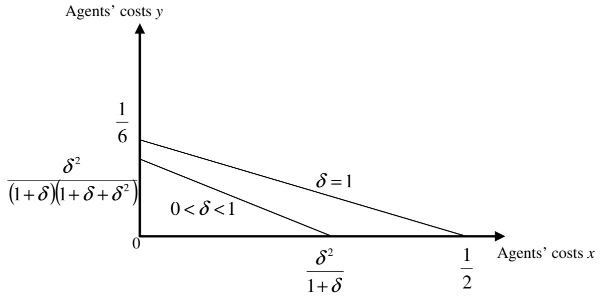


From (2.13), we can work out a relationship between the values of x and y such that bilateral delegation equilibria are possible. We find that:

$$y < \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)} - x\frac{1}{\left(1+\delta+\delta^2\right)} \text{ or } x < \frac{\delta^2}{(1+\delta)} - y\left(1+\delta+\delta^2\right)$$

The first of these relationships is graphically shown in Graph 2.12 for different values of δ. The higher δ, the bigger the range of values of x and y which can support bilateral delegation equilibria. The intercepts with the axes are already known results: when $x = 0$, the maximum level of exogenous costs supporting bilateral delegation equilibria is $y = \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}$. When $y = 0$, the maximum level of endogenous costs is $x = \frac{\delta^2}{1+\delta}$.

Graph 2.12: Costs, impatience and equilibria (II)



2.6 Conclusions

In this chapter we present some models of bilateral delegation. Delegation is introduced as a device to increase one party's bargaining power and we are interested in understanding why, when and how it is sustainable as an equilibrium strategy. Indeed, previous studies showed that one-sided delegation is very effective. With bilateral delegation the analysis is more complicated.

There are four subjects involved in the games: a seller S, a buyer B, and their agents, I and J respectively. These agents are hired by the principals through take-it-or-leave-it offers about the level of some compensation schemes, and then play a Rubinstein game. The structure of the game implies that the agents' profits depend upon both principals' proposals. The first consequence is that the initial Rubinstein game among principals becomes a typical Nash demand game. This means that, with bilateral delegation, the typical first mover advantage disappears.

When delegation is costless, multiplicity arises and uncertainty about the identity of the winner is present. From a positive point of view, this confirms that when delegation is available, the players decide to use it. When a party delegates, the other always replies by delegating as well (a part from extreme cases). Thus, delegating is a (weakly) dominant strategy for both the principals. This is true in a lot of real life situations: workers' unions often negotiate with firms' associations; foreign ministers deal with others foreign ministers, a divorcing couple usually leaves the process to two lawyers. Of course, in many cases delegation is better explained by different reasons. The possibility of saving time, for instance. Or the need of particular skills (i.e., deep legal knowledge). Still, are we sure that a divorcing lawyer would not hire his own lawyer? To stress the importance of the strategic power of delegation, we assume that all the players have the same degree of impatience (which is the only distinctive element of the game).

As one-sided delegation is very powerful, bilateral delegation provides a way of compensating bargaining powers. This compensation is even more important, as it offsets the typical first mover's advantage of Rubinstein games. Delegation is more likely to be profitable for the buyer (or, in general, for the second mover in the original Rubinstein game). Indeed, he has less to lose by switching from the direct bargaining game to the delegation one.

Unfortunately, the model cannot indicate any particular equilibrium to be played. In order to reduce the range of possible equilibria arising, we introduce different kinds of delegation costs, such as principals' exogenous costs (e.g., time consumption for searching an agent), agents' exogenous costs (e.g., training costs) and finally agents' endogenous costs (e.g., income from other possible jobs). A common result is the possibility of prisoner's dilemma type games. The principals may decide to delegate even if they are both worse off with respect to a direct bargaining situation. As usual, the problem lies in the fact that the parties do not collaborate and in the fact that the hiring choice is unknown before the bargaining stage.

When delegation costs are exogenous, agents' costs are more restrictive than principals' ones. This is due to the fact that the latter are time independent and supported on individual basis.

Finally, when we consider endogenous costs, inefficiency concerns are less obvious. These costs are also a source of income and the more the players are patient, the wider the range of bilateral delegation equilibria.

Further developments are possible and, to some extent, necessary. As we note in the introduction, we fail to consider important issues. A more realistic model should consider the presence of incomplete information: compensation schemes or the details of the delegation contracts are not always public. Moreover, the assumption of full bargaining power for the principals is quite extreme. A redistribution of power would lead to different payment schemes and therefore to different equilibrium payoffs.

2.7 Appendix: The payoffs matrix in Table 2.2

The first result in this section is taken from Bester and Sakovics (2001). The second result is obtained following the same logic.

a) When only the seller is delegating, we have the following situation. The seller is delegating the bargaining process to his agent I, and this agent bargains with the buyer. Assuming stationary preferences, in each sub-period the bargaining parties I and B respectively face the following situation:

$$\begin{cases} p_B - f_S = \delta\left(p_I - f_S\right) \\ -p_I + 1 = \delta\left(-p_B + 1\right) \end{cases}$$

which means that, for each player, the gains from accepting the other party's offer today must equal the gains from rejecting it and make an offer tomorrow. Since these gains must be non negative and I moves first, the selling price is:

$$p_I = \frac{1 + \delta f_S}{1 + \delta}$$

The payoffs for each player are: $\Pi_I = p_I - f_S = \dfrac{1 - f_S}{1 + \delta}$; $\Pi_S = f_S$; $\Pi_B = 1 - p_I = \dfrac{\delta}{1 + \delta}(1 - f_S)$. The maximum amount f_S that the agent I will accept is $f_S = 1$, leaving both the agent and the buyer with a zero payoff.

b) When only the buyer is delegating, we have the following situation. The buyer is delegating the bargaining process to his agent J, and this agent bargains with the seller. Assuming stationary preferences, in each sub-period the bargaining parties S

and J respectively face the following situation:

$$\begin{cases} p_J = \delta p_S \\ f_B - p_S = \delta \left(f_B - p_J \right) \end{cases}$$

which means that, for each player, the gains from accepting the other party's offer today must equal the gains from rejecting it and make an offer tomorrow. Since these gains must be non negative and S moves first, the selling price is:

$$p_S = \frac{f_B}{1+\delta}$$

The payoffs for each player are: $\Pi_J = f_B - p_S = \dfrac{\delta f_B}{1+\delta}$; $\Pi_S = p_S = \dfrac{f_B}{1+\delta}$; $\Pi_B = 1 - f_B$. The minimum amount f_B that the agent J will accept is $f_B = 0$, leaving both the agent and the seller with a zero payoff.

Chapter 3

Voting Games with Private Information and Heterogeneous Players

3.1 Introduction

Voting games are often played sequentially. It is not difficult to find examples in real life. The most evident case is probably the presidential election mechanism in the USA: the candidates are locally chosen through primaries and each local primary is set up on a different date[1]. Another example is given by committees. Members often announce their vote, or opinion, in sequence. For instance, in U.S. Navy courts-martial, judges vote in a particular order, leaving the highest ranked ones at the end. In Italy, when expressing

[1] The interested reader can refer to Morton and Williams (1999) for an introduction and an experimental comparison between simultaneous and sequential voting in the United States' presidential primaries.

on particular topics, courts are composed by three members and, according to the code of penal procedure (art. 527), votes are gathered starting from the opinions of the less experienced members (seniority rule).

More specifically, this chapter has been motivated by the observation of an important Italian institution, the "Corte Costituzionale" (Constitutional Court). One of the tasks of this court is to solve questions about the constitutionality of national and regional laws. This duty obviously requires a strong independence from, for instance, the parliament, which actually passes new laws. The fifteen members of this Court are appointed by three different institutional bodies (the president of the Republic, the parliament and high judges). It is fair to assume that members chosen by the parliament might be affected by political bias. We therefore question the rationale of this apparently inefficient choice for an institution which is supposed to pursue social welfare[2].

From a theoretical point of view, the chapter provides insights about the most efficient use of scarce information.

One side of the problem is about the behaviour of uninformed members when they are forced to vote (e.g., they are not allowed to abstain). They face (and solve) the dilemma of wrongly influencing the outcome of an election. To show this result, we initially explore a series of simple voting games, which differ along several dimensions. First of all, we consider sequential and simultaneous voting games of players with truth purpose. Each member has private information, namely a correct signal about the true state of the world, and wants to maximize the expected (common) value of the election. Then, we change the players' preferences and analyze what happens to their equilibrium strategies when they

[2]Historical and political reasons are also provided (see section 3.5.1).

only want to maximize their own probability of being right (reputational purpose). Finally, we introduce heterogeneity and we study the voting behaviour of a committee with three kinds of players.

Another side of the problem is that, in this heterogeneous committee, one member (the biased one) finds it optimal to destroy his information. Surprisingly, this choice provides uniqueness *and* optimality (given that a certain order of voting is imposed). We analyze a simultaneous and a sequential voting game where three members of a committee have different preferences. We show that, in the latter game, the order of voting is relevant and that optimality can be guaranteed. More surprisingly, it is the presence of a biased member which allows for uniqueness of the optimal equilibrium. The intuition for this result is that the bias provides certainty about the voter's strategy and removes an important (and inefficient) piece of uncertainty from the model.

The chapter is organized as follows. After a review of the literature (section two), in section three we introduce two simple voting games: a simultaneous one and a sequential one. We provide intuitions for the results and discuss the emerging equilibria. In section four, we change the players' preferences and solve the same games accordingly. A discussion about the Italian Constitutional Court introduces section five, where we consider a heterogenous committee at work. In section six, we briefly provide preliminary results and intuitions to some extensions of the basic model. Finally, in section seven we discuss the limits of our model, suggest some possible developments of the research and draw our conclusions[3].

[3]I especially thank Jonathan Thomas, Jozsef Sakovics and John H. Moore for their supervision. The paper has been enriched by the participants' comments during presentations at the Scottish Graduate Programme Annual Conference 2004 in Perth (UK), at the Royal Economic Society Easter School 2004 (Birmingham, UK), at the Second World Congress of the Game Theory Society 2004 (Marseille, France)

3.2 Review of the literature

First contributions analyzing voting behaviours date back to the work of Condorcet more than two centuries ago (*"Essai sur l'application de l'analyse a la probabilite des decisions rendues la pluralite des voix"*, 1785). One of his main results is the Jury Theorem. It states that, in dichotomous elections[4], a large population of identical voters is more likely to select the best alternative than a single individual. In this contest, information is provided by (discrete) signals about the true state of the world. So, a majority rule in elections allows for the best outcome[5]. Still, until last decade, the traditional approach has considered mainly the mathematical and statistical properties of these situations. Only recently, have voting games been analyzed as strategic ones.

Table 3.1 summarizes the main contributions in these areas and we discuss them in the rest of this section.

Simultaneous voting. Austen-Smith and Banks (1996) criticize the sincere voting assumption and show that Condorcet's result still holds with strategic voting. They provide game theoretical foundations for his Jury Theorem and show that naive voting is not always the best strategy. In particular, sincere voting is rational and informative only under majority rule and only if majority rule is the optimal aggregation mechanism. In

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and at the 10^{th} Young Economists Spring Meeting 2005 (Geneve, Switzerland), and during seminars in Edinburgh, Catholic University of Milan and Milan-Bicocca. I also thank Pietro Balduzzi, Ferdinando Colombo, David Gill, Mario Gilli and Santiago Sanchez-Pages for useful discussion and valuable suggestions.

[4] Elections with two opposite alternatives. For instance, when the choice is between rejecting or adopting an option.

[5] The Jury Theorem has been generalized by subsequent studies. For instance, Duggan and Martinelli (1999) allow for a continuum of signals. Myerson (1998) shows that Condorcet's result holds for general probability distributions of the informative signals available to the players. Finally, McLennan (1998) demonstrates the existence and the properties of Nash equilibria with mixed strategies in voting games with common values.

an interesting experiment, Eckel and Holt (1989) try to infer the use of sincere voting and strategic voting in a committee taking binary decisions. They find that sincere voting tends to emerge at the beginning of each experiment and that strategic voting is most likely to occur after some meetings, at least when preferences are stationary.

Table 3.1: Review of the main literature

Topic	**Author**
1) Voting literature:	
Simultaneous voting	Austen-Smith (1990)
	Austen-Smith and Banks (1996)
	Feddersen and Pesendorfer (1996, 1997)
	Doraszelski, Gerardi and Squintani (2001)
Sequential voting	Dekel and Piccione (2000)
	Ottaviani and Sørensen (2001)
2) Herd behaviour:	
	Smith and Sørensen (2000)
	Goeree, Palfrey and Rogers (2003)

The relevance of debating and communication is highlighted by several authors. Austen-Smith (1990) introduces a pre-stage for discussion and explores its effects on the outcome of the election (actually, the setting of an agenda). This pre-stage is modelled as a cheap talk one and voting is still simultaneous. His result is that debate can speed up the legislative process. Doraszelski, Gerardi and Squintani (2001) study the positive role of communication in a committee where the players' preferences are private information. Nevertheless, in our model communication is technologically impossible: players only communicate through their final vote.

Two contributions in this area are particularly relevant for our chapter. Feddersen and Pesendorfer (1996, 1997) analyze how well simultaneous voting elections can aggregate private information. Their results are worth discussing, but the papers are not directly comparable with ours, as they focus on the asymptotic properties of their models (i.e., as the number of agents goes to infinity). In their former work, the authors want to explain the low rates of participation in many elections. They emphasize the similarity between dichotomous elections with asymmetric information and auctions with a common value. In both cases, a player must condition his strategy not only on his private information but also on the fact that he may be decisive (i.e., he is the winner of the auction or the swing voter in the election). In equilibrium, uninformed independent voters, whenever they are indifferent between two options, prefer to abstain and not influence the outcome of the game. Informed independent voters choose according to their private information (a binary signal), whereas partisan voters always choose their favorite option. That is, no player plays a strictly dominated strategy. Mixed strategy equilibria (for the uninformed voters) are also possible. Surprisingly, they only involve a mix between abstention and voting for one candidate and not a mix between choosing alternatively one of the two options.

In the latter contribution, the authors rule out abstention (as we do in our model). They discuss equilibrium strategies when voters play strategically and have different preferences. In this model, the symmetric Nash equilibria involve the informed independent voters playing according to their signal and the partisan ones playing their favourite option. Their main result is that (large) elections are decided by informed players and they therefore satisfy full information equivalence; that is, they successfully aggregate private information. Nevertheless, this aggregation is possible as long as the informational structure is not very complex: with high uncertainty about the distribution of preferences and additional possible choices, the effectiveness of elections is weaker. In our chapter, we confirm these equilibrium strategies for informed independent players and partisan ones; we also find optimal replies for independent uninformed players. But our model deals with a small committee, and uninformed statuses are never revealed. The outcome of the election can be determined even by uninformed voters and therefore information cannot be fully aggregated.

Finally, it should be stressed that our committee's objective is to aggregate private information rather than social preferences. Sah and Stiglitz (1988) study the same problem but assume that members always vote honestly[6]. Dulleck and Friederiszick (2002) analyze cases with socially oriented voters *or* career oriented voters. Their models are not directly comparable to ours, especially as they allow for communication and assume that players are ignorant about the quality of their signals.

Herd behaviour. In the last decade, a growing literature about herd behaviour has developed as well. In economics, herding refers to the loss of information due to imitation of previous players' actions. From an information aggregation point of view, this is clearly inefficient. But eliciting information from the sequential structure may be desirable. So there is a trade off which is worthy of analysis.

Seminal contributions in this area are by Scharfstein and Stein (1990), Banerjee (1992)

[6]See Piketty (1999) for a brief review of recent contributions about the information- aggregation role of political institutions.

and Bikhchandani, Hirshleifer and Welch (henceforth BHW, 1992 and 1998). General models are developed in Banerjee (1992) and BHW (1992). The former is a model where individuals are unevenly informed: some of them observe a signal about the true state of the world and some do not. The signal is correct with a certain probability and can take a continuum of values. The author emphasizes the inefficiency emerging from this situation and the importance of the first individuals' choices. The latter model is different to the extent that players observe binary signals, which are identically distributed among the players. In the same paper and in a following one (BHW, 1998), the authors also discuss the fragility of these informational cascades and apply their model to explain fads and cultural changes. For example, they argue that the presence of more informed people (with more than one signal), new comers (not knowing past actions) or individuals with different preferences may weaken the possibility of cascades (and even stop them). Scharfstein and Stein (1990) present a more specific model, where the probability of observing a signal is conditional to the type (ability) of the specific individual. Gale (1996) challenges the robustness of these results. In particular, he focuses on the nature of signals, possible actions, sequence of the choices, discreteness of time, possibility of cheap talks. Palley (1995) offers a different interpretation of managerial herding, which is due, in his opinion, to the existence of relative performance remuneration schemes. Finally, Swank and Visser (2003) show that, by endogenizing information (information is not free but must be bought), herding disappears.

Though, for our purposes the most relevant references in herding are by Smith and Sørensen (2000) and Goeree, Palfrey and Rogers (2003). They both show how the importance of herding can be weaker when we introduce heterogenous preferences. Goeree, Palfrey and Rogers (2003) analyze the case with both private and common values (i.e., preferences depending on the final outcome of the ballot and on some ideological bias). Smith and Sørensen (2000) consider differences only along the common value dimension. The main assumption in this literature is that other players' actions do not influence each player's own payoffs. We show that this assumption does not necessarily hold in a sequential voting procedure.

Sequential voting. Voting and herd behaviour are both very useful in the analysis of sequential voting games.

Two main papers discuss sequential voting games[7]. Dekel and Piccione (2000) compare simultaneous and sequential voting games where individuals observe a signal about the true value of an option. They then decide whether to accept this option ("yes") or to stay with a *status quo* ("no"). Every player votes after observing his signal and the previous players' choice. More than one individual can vote in each period. Their main result is that both simultaneous and sequential voting games are effective in aggregating information. Moreover, as far as symmetric equilibria are concerned, the sequential structure cannot do better than the simultaneous one. The intuition is not very difficult. Voters are behaving strategically and condition their actions on being pivotal in the election. As long as strategies are symmetric and conditional, knowing who voted before you does not provide any useful information. This result applies to certain asymmetric equilibria as well (but they do not state a general rule). An interesting application is provided by elections with a common value for the alternative; that is, elections whose sole aim is to aggregate

[7] Sloth (1993) introduces and discusses sequential voting games, but only as a device to solve simultaneous ones. She shows that sophisticated equilibria in the former games are equivalent to subgame perfet equilibria in the latter.

private information. In (the unique) equilibrium, information is fully aggregated until an option is chosen; afterwards, the players do not need to vote informatively as they are no longer pivotal. This equilibrium recalls some characteristics of herd behaviour. Of course, this result does not necessary hold in any setting. In an example, the authors show that it is possible to find a better (asymmetric strategies) equilibrium with sequential voting when there are more than two signals. This last point is important and leaves the door open for further research. Our own chapter provides an analysis of cases where the best equilibrium strategy profiles are asymmetric ones. Nevertheless, we are not directly interested in how information is aggregated, and we consider players with heterogeneous preferences. In addition, even in games with homogeneous players, our environment is not totally symmetric: it is not necessarily the case that every player observes a signal.

Ottaviani and Sørensen (2001) consider a game where members of a committee ("experts") sequentially reveal their opinions to a decision maker and finally a decision is taken. Members care about their reputation, but not directly about the final decision. That is, they only want to show (to the market) that they make the right choice. Each of them observes a different signal, but does not know its quality. If the signal is very different from the common prior he shares with the other players, than he prefers to pool and play uninformatively. This loss of information parallels the findings of herding literature. The authors also try to endogenize the order of speech to find an optimal one. In general, better experts should speak first. As herding arises, some information may be lost. This is especially true with more than five members and with greater heterogeneity in the quality of the experts. With experts of unknown ability, there are equilibria in simultaneous games dominating equilibria in sequential ones. The opposite is true with known ability.

Our chapter may be viewed as a particular case where all the voters have the same known ability. We introduce heterogeneity in terms of preferences (but not ability) and find that the optimal order of voting actually depends on them.

3.3 Voting games with truth purpose

In this section, we study two voting games, both based on the following setting.

A committee must take a binary decision: it can accept an alternative (voting "yes") or reject it (voting "no"). Suppose the number of members, n, to be 3 and the required majority to be 2. The *status quo* has a common value of 0. For any member m of the committee (with $m \in M = \{i, j, z\}$), the alternative can take two common values: $v_m \in \{1; -1\}$. Each value has the same prior (i.e.,$\frac{1}{2}$) but a private signal about the true one can be observed with probability α. As we are dealing with "experts", we assume $\alpha \in [\frac{1}{2}, 1]$[8]. When observed, the signal is correct with probability β. For simplicity, we fix $\beta = 1$[9]. Every player expresses a vote and abstention is not possible. A strategy consists in a member's voting behaviour. The usual notation applies: s_m is the generic player m's strategy and s_{-m} is everyone else's strategies. In this section, we assume that the players are mainly interested in the committee taking the right decision (*truth purpose*). Nevertheless, they are not willing to reveal an uninformed status to their fellow committee members, so any information transmission occurs solely through voting behaviour (i.e., there is no pre-voting debate). We may think of this case as a situation where the public (*rectius*, who appoints the committee) observes only the final decision.

[8] This assumption is made without loss of generality.
[9] See section 3.6 for preliminary results when $\beta < 1$.

We now define optimality in our context.

Definition 2 (Optimality) *A generic player m's strategy s_m is individually optimal (s_m^*) when it maximizes the player m's own utility function, $u_m(s_m \mid s_{-m})$:*

$$s_m^* : \arg\max u_m(s_m \mid s_{-m})$$

A strategy profile $s = (s_i, s_j, s_z)$ is socially optimal (s^*) when it maximizes the probability that the committee takes the right decision (equivalently, the expected value of the election) given the available signals:

$$s^* : \arg\max E(v_m)$$

We need Definition 2 to stress that optimality is here defined as a constrained concept. Due to lack of communication, full information aggregation is not always possible. This means that for members with truth purpose, individual and social optimum coincide. We will refer to our former constrained concept simply as *optimality* and to the latter as *full information optimality.* We say that the concept is constrained because with full information the probability of taking the right decision is always higher[10].

The probability of taking the right decision in the simultaneous voting game with truth purpose is therefore the benchmark case we will refer to through the chapter when discussing the efficiency implications of different committee compositions or voting mechanisms.

At this point, it is worth noting the difference between the herding literature and this case. Herd behaviour and cascades are a consequence of optimizing behaviour of agents,

[10] See footnote 13.

whose utility is independent from the followers' actions. They find it optimal to ignore their private information as they are not concerned with its effects on subsequent players.

Now, is this still true in voting games with truth purpose? Consider a committee of three members as described above where votes occur sequentially. The first and third players play the following strategy: play the signal if observed, randomize if not. Then, assume the first chooses "yes". If the second player is uninformed, he assigns a probability $\alpha + \frac{1}{2}(1-\alpha) > \frac{1}{2}$ to "yes" being the true state. Indeed, he thinks that the first player voted "yes" either because he was informed (probability α) or because he randomized as uninformed (probability $1-\alpha$). We may think that the second player should follow, as the probability of being right by imitating (left-hand side of the previous inequality) is higher than by randomizing (right-hand side). Is this our conclusion? If so, we would have a typical result of herd behaviour. Well, this is not necessarily the case. He may optimize as well by doing exactly the opposite! In this way, the responsibility of the right decision is given to the third player, whose probability of being right is exactly the same as the first one: $\alpha + \frac{1}{2}(1-\alpha)$.

Herding is not a straightforward conclusion in this sequential voting game. The reason is that a player's utility is no longer independent of the followers' actions.

Bearing in mind this example, we can now analyze two types of voting games: a simultaneous one and a sequential one.

3.3.1 The simultaneous voting game

Two signals about the true state of the world are possible: $\omega_m = \{H; L\}$, and each player observes a signal with known and common probability $\alpha \in [\frac{1}{2}, 1]$[11]. If the true value of the alternative is 1(alternatively, -1), then the only possible signal is $H(L)$. With a little abuse of notation, we say that the information set of the generic player m is simply $\Omega_m = \{\omega_m\}$ when he is informed, as he perfectly knows the true state of the world. This is indeed equal to the signal he observes. On the contrary, when he is uninformed his information set is $\Omega_m = \{H, L\}$, as he does not know the true state of the world.

We also assume that players condition their strategy on being pivotal. Since any strategy is optimal when the player is not pivotal, we concentrate on weakly dominant strategies.

Proposition 5 introduces the first (and only) symmetric equilibrium of the game.

Proposition 5 *The simultaneous voting game with truth purpose has only one symmetric equilibrium. In this equilibrium, the generic player m's strategy s_m is:*

$$s_m = \left\{ \begin{array}{l} \textit{play } \omega_m \textit{ if } \Omega_m = \{\omega_m\}; \\ \\ \textit{randomize } 50:50 \textit{ if } \Omega_m = \{H, L\} \end{array} \right\}$$

Individual optimality requires that every player plays his signal if observed, as this is right with probability $\beta = 1$. What happens if the signal is not observed is not straightforward. In the symmetric equilibrium, the uninformed player i is indifferent between

[11] A model with three signals, where in each state the correct signal is observed with probability α and an uninformative one with probability $1 - \alpha$, is isomorphic to this one.

accepting or rejecting the alternative. He also knows that:

$$s_{-i} = \left\{ \begin{array}{l} \text{play } \omega_{-i} \text{ if } \Omega_{-i} = \{\omega_{-i}\}; \\ \\ \text{randomize } 50:50 \text{ if } \Omega_{-i} = \{H, L\} \end{array} \right\}, \text{ where } \ -i = j, z$$

As $\alpha \geq \frac{1}{2}$, i knows that it is very likely that at least one of the other two players observed a signal, so he should follow that player. Unfortunately, this is not possible in a simultaneous game. As i is conditioning his strategy on being pivotal, he assumes that j and z played the opposite. Given their strategy, i is indifferent between following j or z. The best i can do is randomizing.

Nevertheless, this is not necessarily the only and the best possible equilibrium strategy profile. In Appendix (section 8), we prove that there are several better equilibria if we allow for asymmetric strategies.

Consider the following strategy:

$$s_{i(j,z)} : \left\{ \begin{array}{c} \text{play } \omega_{i(j,z)} \text{ if } \Omega_{i(j,z)} = \{\omega_{i(j,z)}\}; \\ \\ \text{play "yes" with probability } p(q,r) \text{ if } \Omega_{i(j,z)} = \{H, L\} \end{array} \right\}$$

that is, playing the signal if observed and playing "yes" with some probability if not. As we argued before, the first part of the strategy must be optimal, as the signal is always correct. For the second, we must solve for the utility maximizing probabilities. Before introducing and discussing Proposition 6, which characterizes the set of equilibria of this game, we need Definition 3.

Definition 3 (Compensation) *Two players are compensating each other when the following two conditions are satisfied:*

- *they are both uninformed;*

- *they play "yes" with probabilities whose sum is equal to 1. When these probabilities take extreme values (i.e., 0 or 1), then their actions involve "compensation in pure strategies".*

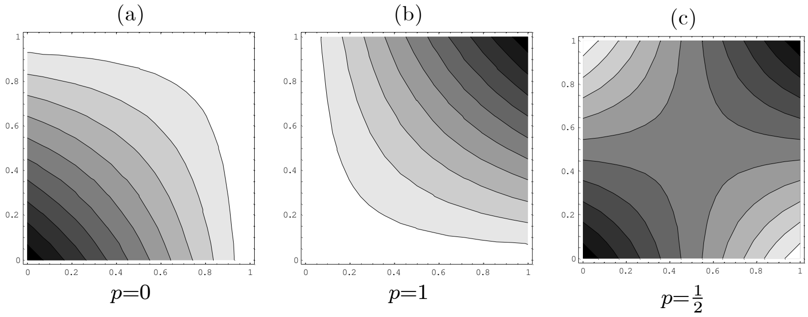
Proposition 6 *The simultaneous voting game with truth purpose has a continuum of equilibria in weakly dominant strategies. In any of the equilibrium strategy profiles, every member plays according to the signal, if any, and plays "yes" with the following probabilities if not:*

$$\begin{aligned}
\{p, q, r &\in [0,1]^3 : q + r = 1, qr = 0\}, \\
\{p, q, r &\in [0,1]^3 : p + q = 1, pq = 0\}, \\
\{p, q, r &\in [0,1]^3 : p + r = 1, pr = 0\}, \\
\{p, q, r &\in [0,1]^3 : p = q = r = \frac{1}{2}\}.
\end{aligned}$$

The socially optimal equilibrium strategies are asymmetric ones and involve compensation in pure strategies.

We have already discussed the logic behind the internal solution. Using Definition 3, we can state that there is only one equilibrium with compensation in mixed strategies and it does not maximize social welfare as we have defined it. The corner solutions are equilibria with compensation in pure strategies. In Graph 3.1, we show the ranges of possible equilibria for fixed values of p. On the horizontal axis we measure q and on the vertical one r.

Graph 3.1: Equilibrium strategies in the simultaneous voting game



In the contour graphs, the lighter the colour the higher the value of the represented function (in this case, the committee's expected value). As we have shown, for the internal value $p = \frac{1}{2}$, an equilibrium is reached when $q = r = \frac{1}{2}$. Nevertheless, it evident from Graph 3.1c that this is only a saddle point, so it is not optimal. For extreme values of p (Graphs 3.1a and 3.1b), the maximum is reached by fixing either q or r in order to compensate p, leaving the other probability free to change in the set $[0, 1]$. For every internal value of p (i.e., Graph 3.1c), q and r must compensate each other at extreme values (i.e., $q = 0; r = 1$) to maximize the social welfare.

Given $\alpha = \frac{1}{2}$, the maximum possible expected value of the election is $E(v_m) = \frac{3}{8}$. In Graph 3.1c, this level is reached in all the combinations of q and r represented by a white area, that is $q = 0, r = 1$ and $q = 1, r = 0$.[12].

As long as individuals are uninformed, at least two of the players should compensate each other in order to minimize their influence on the outcome of the game. This compensation reaches the social optimum when it is played in pure strategies; that is, corner solutions dominate the symmetric one. For $p = q = r = \alpha = \frac{1}{2}$, the expected value of the

[12] Unfortunately, some confusion may arise as the borders of the graphs are white as well.

election is indeed only $\frac{11}{32} < \frac{3}{8}$.

The difference between full information aggregation and our constrained concept of optimality can be effectively addressed here. It is probably better understood with an example. Suppose players i, j and z are playing the (constrained) optimal equilibrium strategy profile $s^* = \{s_i^*, s_j^*, s_z^*\}$, where:

$$s_i^* \; : \; \left\{ \begin{array}{c} \text{play according to the signal if observed;} \\ \text{always play "no" if uninformed (i.e., } p = 0) \end{array} \right\};$$
$$s_j^* \; : \; \left\{ \begin{array}{c} \text{play according to the signal if observed;} \\ \text{always play "yes" if uninformed (i.e., } q = 1) \end{array} \right\};$$
$$s_z^* \; : \; \left\{ \begin{array}{c} \text{play according to the signal if observed;} \\ \text{always play "no" if uninformed (i.e., } r = 0) \end{array} \right\}.$$

Conditional on L being the true state, the decision is always correct. Conditional on H being the true state, the final decision may be wrong in two cases: either no one is informed, or only j is informed. Both the cases have a probability of $\frac{1}{8}$ when $\alpha = \frac{1}{2}$ and the expected value of the election, as already stated, is $\frac{3}{8}$. If we could fully aggregate information, then the committee would be wrong only when no one was informed, as j could share his signal with the others. With full information aggregation, the expected value of the election would be $\frac{7}{16} > \frac{3}{8}$[13].

We believe that this compensation result has some similarities to the one in Feddersen and Pesendorfer (1996). In that case, the optimal strategy for uninformed members was to

[13] See Appendix for a proof and a derivation of the expected value of the election with full information aggregation.

abstain. In our model, abstention is not possible but can be mimicked by compensation. The problem with compensation is that it may prevent full aggregation of information, as an uninformed player, when compensating, might in fact cancel out the vote of an informed player.

3.3.2 The sequential voting game

The model we develop in this subsection is slightly more complex. Followers (can) have different actions according to different histories of the game. This also means that the information set of the generic player m, Ω_m, contains the signal, if observed, and the voting history. The voting history is the collection of voting decisions d, expressed by members who have already played. Given an order of vote (e.g., i, j, z), we characterize the players' strategies as follows:

$$s_i : \left\{ \begin{array}{c} \text{play } \omega_i \text{ if } \Omega_i = \{\omega_i\}; \\ \text{play "yes" with probability } p \text{ if } \Omega_i = \{H, L\} \end{array} \right\},$$

as the first voter is in the same situation as in the simultaneous game;

$$s_j : \left\{ \begin{array}{c} \text{play } \omega_j \text{ if } \Omega_j = \{\omega_j, d_i\}; \\ \text{if } \Omega_j = \{H, L; d_i\}\text{:} \\ \text{play "yes" with probability } q_Y \text{ when } d_i = \text{"yes"} \\ \text{play "yes" with probability } q_N \text{ when } d_i = \text{"no"} \end{array} \right\},$$

that is, j votes according to his signal if observed, as this is always correct, independently on the vote cast by i. When j does not observe a signal, he instead conditions his vote on

i's decision. Finally:

$$s_z : \left\{ \begin{array}{c} \text{play } \omega_z \text{ if } \Omega_z = \{\omega_z, d_i, d_j\} \\ \text{if } \Omega_z = \{H, L; d_i, d_j\}\text{:} \\ \text{play “yes” with probability } r_{YY} \text{ when } d_i = d_j = \text{“yes”} \\ \text{play “yes” with probability } r_{YN} \text{ when } d_i = \text{“yes” and } d_j = \text{“no”} \\ \text{play “yes” with probability } r_{NY} \text{ when } d_i = \text{“no” and } d_j = \text{“yes”} \\ \text{play “yes” with probability } r_{NN} \text{ when } d_i = d_j = \text{“no”} \end{array} \right\},$$

which means that z always follows his signal, if any, and conditions his vote to the voting history when uninformed. Each player's strategy is no longer independent from previous voters' ones. From Dekel and Piccione (2000), we know that any symmetric equilibrium in the simultaneous voting game is an equilibrium in the sequential one. Hence, independently of history, playing "yes" with probability $\frac{1}{2}$ when uninformed is still an optimal strategy for each player. But the best equilibria we are dealing with are asymmetric ones. We are therefore interested in understanding whether they can be replicated in this different game and, above all, whether a better equilibrium strategy profile can be found.

The game is solved as before, but with the presence of more variables (probabilities). The analysis is simplified if we note that r_{YY} and r_{NN} are irrelevant, as a decision has already been taken.

The results are presented in Proposition 7, which is then discussed (and proved in the Appendix).

Proposition 7 *Every equilibrium strategy profile of the simultaneous game is also an equilibrium in the sequential one, but more equilibria are sustainable in the latter. These addi-*

tional equilibria (in weakly dominant strategies) still involve compensation in pure strategies and are optimal. They are characterized by the following properties: when informed, members vote according to their signal; when uninformed, they play "yes" with probabilities satisfying the following conditions:

$$\left\{\begin{array}{c} r_{YN} = r_{NY} = 1 - p; \\ \\ pr_{YN} = 0; \\ \\ \forall q_Y, q_N \in [0,1] \end{array}\right\} \; or \left\{\begin{array}{c} q_N = q_Y = 1 - p; \\ \\ pq_Y = 0; \\ \\ \forall r_{YN}, r_{NY} \in [0,1] \end{array}\right\}$$

in the first type of equilibrium profile, and:

$$\left\{\begin{array}{c} r_{YN} = 1 - q_Y \neq r_{NY} = 1 - q_N; \\ \\ r_{YN}q_Y = 0; r_{NY}q_N = 0; \\ \\ \forall p \in [0,1] \end{array}\right\}$$

in the second one. The maximum expected value of the election is the same as in the simultaneous game.

Both of the new equilibria are corner solutions. The first new equilibrium is a straightforward extension of the previous one: once two players compensate each other, it does not make any difference whether the third is discriminating (as in the sequential game) or not (simultaneous game).

The last equilibrium profile is the most different one, and all the players differentiate their actions. The intuition behind this last case is the following. Suppose $q_Y = 1$: this means that, after the first player i votes "yes", the second player j always plays "yes" if uninformed. Then, the third player z knows that, if he observes j voting "no", then it must be case that j is informed and "no" is the correct choice. The optimal answer for z is playing $r_{YN} = 0$. When $q_Y = 0$, then the optimal reply is $r_{YN} = 1$. Now z can not infer from observing j voting "no" whether he is informed or not: so the best he can do is compensating when uninformed. Of course, compensation is socially bad if j is actually informed. It is exactly the possibility of this information destruction which precludes full aggregation of information. A similar argument explains the relation between r_{NY} and q_N.

The range of equilibria is wider than in the simultaneous game. Compensation is still a condition, but this may occur in more complex ways. For instance, a compensation conditional on the voting history was involved in the last equilibrium strategy profile we discussed.

Quite surprisingly, the possibility of discriminating the action according to the history does not provide any improvement from a social welfare point of view. This means that the sequential structure does not add anything to the information aggregation process. The only gain is that coordination appears to be more credible when players vote sequentially.

3.4 Voting games with reputational purpose

Suppose now the members in the same committee are only interested in appearing informed. They do not care about the final decision of the committee but want to show that they are right, even when they are uninformed. In this context, "being right" means voting according to the true state of the world. That is, voting "yes" when $v_m = 1$ and voting "no" when $v_m = -1$. The opposite is true for "being wrong".

In this game, voters care about their own reputation (*reputational purpose*). A tension with the socially optimal behaviour might therefore emerge.

The rest of the model does not change: the players still face two states of the world and can observe one of the two possible signals with probability $\alpha \in [\frac{1}{2}, 1]$.

The new utility function of the generic member m is the following

$$u_m = \begin{cases} 1 & \text{when } m \text{ is right} \\ 0 & \text{when } m \text{ is wrong} \end{cases}$$

In this section we solve again a simultaneous and a sequential game, showing how the equilibrium strategies depend on the agents' utility functions.

3.4.1 The simultaneous voting game

A player's strategy no longer needs to be conditional on the fact that he is pivotal. Even though he knows he cannot influence the committee's final decision, a player is still maximizing his expected utility function.

Suppose m plays the usual generic strategy:

$$s_m : \left\{ \begin{array}{c} \text{play } \omega_m \text{ if } \Omega_m = \{\omega_m\}; \\ \text{play ``yes'' with probability } p \text{ if } \Omega_m = \{H, L\} \end{array} \right\}$$

that is, playing the signal if observed and playing "yes" with some probability if not. Then, it is straightforward to work out that m's expected utility is:

$$E(u_m) = \frac{1+\alpha}{2}$$

which is independent of the probability p. So, Proposition 8 follows.

Proposition 8 *Any p is utility maximizing for the generic player m. His optimal strategy s_m^* is:*

$$s_m^* : \left\{ \begin{array}{c} \textit{play } \omega_m \textit{ if } \Omega_m = \{\omega_m\}; \\ \textit{play ``yes'' with any probability } p \in [0,1] \textit{ if } \Omega_m = \{H, L\} \end{array} \right\}$$

The solutions found in the game of section 3.3.1 are only particular cases of these ones. In this game any probability p, q and r is an equilibrium strategy, so the social welfare is not necessarily maximized. We recall that social optimality requires compensation in pure strategies.

3.4.2 The sequential voting game

In the sequential version of this game, the first player is in the same situation as in the previous subsection. We still assume that members vote in this order: i, j, z.

So, player i plays "yes" with any $p \in [0,1]$ and is right with probability $\frac{1+\alpha}{2}$.

When uninformed, the second player should play as the first one, as $\frac{1+\alpha}{2} > \frac{1}{2}$. The left hand side of the inequality is the probability that the i is right, and the right hand side is the probability that j is right by randomizing. If informed, then j should play according to the signal.

The third player z faces two possible cases: i and j played the same or they played the opposite. As long as z has a signal, the difference does not matter; he knows his signal is always correct and he plays it. But if he is uninformed, then he should follow j. Indeed, if j and i played the same, this is straightforward, because z is observing two voters doing the same. But if they did not, then j played the opposite only because he was informed. So he must be followed.

According to our discussion, Proposition 9 holds.

Proposition 9 *The individually optimal strategies of this game are:*

$$s_i^* : \left\{ \begin{array}{c} \textit{play } \omega_i \textit{ if } \Omega_i = \{\omega_i\}; \\ \\ \textit{play "yes" with any probability } p \in [0,1] \textit{ if } \Omega_i = \{H, L\} \end{array} \right\}$$

$$s_j^* : \left\{ \begin{array}{c} \textit{play } \omega_j \textit{ if } \Omega_j = \{\omega_j, d_i\}; \\ \\ \textit{play as the previous player if } \Omega_j = \{H, L; d_i\} \end{array} \right\}$$

$$s_z^* : \left\{ \begin{array}{c} \textit{play } \omega_z \textit{ if } \Omega_z = \{\omega_z, d_i, d_j\}; \\ \\ \textit{play as the previous player if } \Omega_z = \{H, L; d_i, d_j\} \end{array} \right\}$$

This equilibrium is clearly different from the ones above. As soon as an individual is uninformed, imitation arises. We cannot really talk of herding, since no information is lost in the process[14].

In the next subsection, we compare all these results and comment on our findings.

3.4.3 Comments

In Table 3.2, we report equilibrium strategies of the games we analyzed. In all of them, there is a common action: "play the signal if any". For this reason, we focus only on the actions a player should take in equilibrium when he is uninformed. The table is bi-dimensional. On the horizontal line we discriminate by the utility function of the agents: the first column represents agents caring about the correct choice by the committee and the second column represents agents caring about their reputation. On the vertical dimension,

[14] See Definition 4 below.

we consider the different voting games: the simultaneous and the sequential.

In both of the voting games with a common truth interest, herding is not a consequence. All the equilibria involve compensating behaviour: a destruction of possible cascades. The idea behind this result is that no uninformed member wishes to overly influence the final outcome of the game. As abstention is not possible, the only way to reach this result is through some coordination. As we pointed out, there are several equilibria in truth purpose games and the best ones involve pure strategy for two of the players.

Table 3.2: Equilibrium strategies

	Truth	Reputation
Simultaneous	(game A) Compensation	(game C) Any
Sequential	(game B) Compensation	(game D) Imitation

On the contrary, in voting games with reputational concerns, a difference emerges between the sequential and the simultaneous structures. The former weakly dominates the latter in social welfare terms.

The expected value of the election in game D is given by the function:

$$E(v_m) = \frac{1}{2}\left[Y(\cdot \mid v_m = 1) - Y(\cdot \mid v_m = -1)\right],$$

where the function $Y(\cdot)$ represents the probability that "yes" wins when either $v_m = 1$ or $v_m = -1$. In particular:

$$Y(p \quad | \quad v_m = 1) = [\alpha + (1-\alpha)p] + (1-\alpha)(1-p)\alpha$$

$$Y(p \quad | \quad v_m = -1) = (1-\alpha)^3 p + (1-\alpha)^2 \alpha p$$

When $v_m = 1$, the committee votes "yes" whenever the first player votes "yes", or when he does not but the second player is informed. When $v_m = -1$, the committee votes "yes" when nobody (or only the third player) is informed and the first votes "yes". For $\alpha = \frac{1}{2}$, we have:

$$E(v_m) = \frac{3}{8}, \quad \forall p \in [0, 1]$$

which is the maximum value of the function for this particular value of α.

Every equilibrium in game D is socially optimal but only a few of the multiple equilibria in game C are. When $\beta = 1$ and players have reputational purposes, then imitation arises as an efficient behaviour in the sequential game. The existence of equilibria which are all optimal is quite surprising as we are dealing with a game where social welfare is not the objective of the players. But this is not exact: actually, social welfare is not *necessarily* the objective of the players. When it is compatible with their individual aims, then it can be a result, as in this case.

3.5 Voting games with heterogeneous players

So far, we have focused on homogenous players: they all have the same ability (i.e., the probability that their signal is correct is the same) and the same preferences. We now introduce heterogeneity. In this context, heterogeneity can have two dimensions: on the one hand, it may concern the ability of the players; on the other hand, it may regard their objective functions. The former type of heterogeneity has been already widely analyzed (see, for example, Ottaviani and Sørensen, 2001). Quite surprisingly, to our knowledge no paper deals with the possibility of different preferences in sequential voting games

(Feddersen and Pesendorfer only deal with it in simultaneous games). We believe that the order of vote is relevant in the sequential game. This is one of the reasons for deciding to concentrate on this point.

The other reason is the fact that, in real life, we may expect to observe committees where members do not share exactly the same preferences. One possible example is the Italian Constitutional Court. According to the Italian Constitution (art. 135 catch 1), the 15 members of this court are experienced judges, one third of whom are appointed by the President of the Republic, one third by high levels of the magistrature and the remaining one third by the Parliament. It is not so strange to imagine that each of them could have, in principle, different preferences. We could suppose that those appointed by the President are totally independent; those appointed by magistrates are mainly status seekers and, finally, those appointed by the Parliament are policy biased, in order to favour the party which sustained them. This assumption about types is very strong. We do not actually believe that a biased member is necessarily and exclusively concerned with satisfying his political side, nor that the status seeker has no interest in social welfare, nor that the truth seeker has no political opinions. A more accurate model would weight each of these three components for each of the members, but we reckon that this additional complexity would not add relevant insights to the current model.

Before introducing our main model, we devote a subsection to the study of this Italian institution, trying to understand historical and political reasons for its peculiar composition. Our model will then provide game theoretic foundations for it.

3.5.1 The Italian Constitutional Court

The Constitutional Court has been introduced in the Italian legal and political system by the Constitution of the Republic. This document was approved at the end of 1947 by the Constitutional Assembly, on the basis of a proposal made by a subcommittee, called the Committee of the 75s. The Court has existed only since 1955 and its first decision dates back to 1956.

Articles 134-137 of the Constitution are devoted to its organization; however, we are only interested in the first two of them. According to art. 134, the Court is in charge of solving:

- questions about the constitutionality of national and regional laws;
- conflicts among bodies of the State, among State and regions and among regions themselves;
- accusations against the President of the Republic.

In addition, the Court decides whether requests of referenda are admissible (constitutional law no. 1/53, art. 2). The duties of the Court are extremely important, as they strongly influenced the debate about its composition[15], which is our main interest here.

Indeed, two main opinions emerged in the Constitutional Assembly: for some members, the Court was mainly a political body and therefore needed to be controlled by a political organization. These people thought that the Parliament should have been in charge of appointing all its members and that membership itself should end after new political elections. For other members, the Court was basically a legal body; in particular, it should

[15] See Falzone, Palermo and Cosentino (1976).

be independent of the political power exactly for its duty as "judge" of the constitutional correctness of Parliamentary decisions.

The proposal presented by the 75s (art. 127 of the proposal) stated that half of the Court should be composed of judges, a quarter of lawyers and professors and a quarter of citizens. All of them would have been appointed by the Parliament, who would choose from a list of names (three times the required number of members), and would have been in office for nine years.

In the Assembly, the latter position was dominant and some catches were changed. In particular, according to art. 135 of the approved Constitution, the Court is composed of fifteen members, chosen only from judges (even if retired), lawyers with more than 20 years of experience and professors. Among these fifteen members:

- five are appointed by the President of the Republic;
- five by the Parliament;
- five by the highest judges.

As we can see, the political control has become milder, especially in that the office of a member lasts nine years and that of an MP no more than five (unless he is re-elected).

The final decision of the Assembly was in favour of the "jurisdictional" view of the Court. Despite this, and in line with the strong compromise behaviour of its members, some political control has been guaranteed to the Parliament. The desire for a balance between the political and jurisdictional duties of the court was eventually realized.

Whether this choice is inefficient from a social welfare point of view is the question we wish to answer in the rest of this work.

3.5.2 A model of voting games with heterogeneous players

In an attempt to model the composition of the Italian "Corte Costituzionale", we study the behaviour of a committee with heterogeneous players. For simplicity, we still assume $\beta = 1$ and the existence of the following three members. The first is truth-seeking, a social optimizer chosen by the president of the Republic; the second is status-seeking, mainly concerned about his reputation among other judges, who elected him. The third is policy biased, interested in satisfying the party who supported him. We label them as I (independent, with truth purpose), R (status seeker, with reputation purpose) and P (policy biased) respectively. The first two players' objective functions are already known:

$$u_I : E(v)$$

that is, the independent member maximizes the expected (common) value of the election; and:

$$u_R = \begin{cases} 1 & \text{when } R \text{ takes the right decision} \\ 0 & \text{when } R \text{ takes the wrong decision} \end{cases}$$

Additional specifications about P are necessary. His bias b can take two values: $b : \{Y, N\}$. We call him *conservative* if he is biased towards the *status quo* option ($b_P = N$), and *reformist* when he prefers the alternative ($b_P = Y$). He always supports his political opinion, systematically ignoring his private information. His utility is a function $u(o)$ of the final outcome of the game, o. When the alternative passes ("yes" wins), then $o = Y$; when the alternative loses ("no" wins), then we have $o = N$. Accordingly, $u(o)$ can assume two values. Suppose P is a reformist; then $u(Y) = 1$ and $u(N) = 0$. On the contrary, if he is a conservative, then $u(Y)=0$ and $u(N)=1$. To summarize:

$$u_P : \begin{cases} 1 & \text{when } o = b \\ \\ 0 & \text{when } o \neq b \end{cases}$$

Preferences are common knowledge among the players and we assume P to be a reformist.

It is worth stressing, especially if the reader thinks in terms of the example provided, that uninformativeness in this case should be interpreted as noisiness of available information. For instance, a judge is said to be uninformed when there are strong arguments supporting both the possibilities (rejecting or accepting).

The remaining assumptions of the model remain the same.

The simultaneous voting game

We first present the main result of the game and then we illustrate it.

Proposition 10 *In the simultaneous voting game with three heterogeneous players, multiple equilibria (in weakly dominant strategies) arise, including a unique socially optimal equilibrium.*

We now prove the Proposition 10. Suppose P is a reformist. Then he has a simple dominant strategy, which is:

$$s_P^* : \{\text{always play “yes”}\},$$

and maximizes his payoff function.

As for R, we know from previous discussion that in the simultaneous voting game his choice is independent from the other players' choices. So his utility maximizing strategy is still:

$$s_R^* : \left\{ \begin{array}{c} \text{play } \omega_R \text{ if } \Omega_R = \{\omega_R\}; \\ \\ \text{play "yes" with any probability } q \in [0,1] \text{ if } \Omega_R = \{H, L\} \end{array} \right\}$$

Finally, the independent player's equilibrium strategy must look like the following:

$$s_I : \left\{ \begin{array}{c} \text{play } \omega_I \text{ if } \Omega_I = \{\omega_I\}; \\ \\ \text{play "yes" with probability } p \text{ if if } \Omega_I = \{H, L\} \end{array} \right\},$$

as the signal is always correct. The optimal p can be easily found: player's I utility function is given by:

$$u_I : E(v) = \frac{1}{2} \left[Y(p, q \mid v = 1) - Y(p, q \mid v = -1) \right]$$

where $Y(\cdot)$ is the probability that "yes" wins. P's strategy is unique and certain (P always votes "yes"), so u_I only depends upon q and p. In particular, the expected value of the election is negatively related to both p and q. When $v = 1$, the committee expresses a positive vote when at least I or R is informed or, if uninformed, at least one of them votes "yes":

$$\begin{aligned} Y(p, q \quad | \quad & v = 1) : \\ & [\alpha + p(1-\alpha)][\alpha + q(1-\alpha)] + \\ & [\alpha + p(1-\alpha)](1-q)(1-\alpha) + \\ & (1-p)(1-\alpha)[\alpha + q(1-\alpha)] \end{aligned}$$

When $v = -1$, the committee expresses a positive vote when at least I or R is uninformed and votes "yes":

$$
\begin{aligned}
Y(p, q \quad | \quad & v = -1) : \\
& (1-\alpha)^2 pq + \\
& (1-\alpha)p[(\alpha + (1-q)(1-\alpha)] + \\
& (1-\alpha)q[(\alpha + (1-p)(1-\alpha)]
\end{aligned}
$$

We obtain:

$$u_I : E(v) = \frac{1}{2}\left[\alpha p(\alpha - 1) + \alpha q(\alpha - 1) + \alpha(2-\alpha)\right]$$

The utility maximizing level of p must be 0. I cannot choose q, so an equilibrium does not need to be optimal. There is only one socially optimal equilibrium which is reached when R plays $q = 0$ as well.

In other words, given P's dominant strategy, I's best reply is compensation in pure strategies, as in the truth purpose game. R's strategy is independent from the others', so in general we should not expect a socially optimal equilibrium profile to be selected.

The result is striking in the sense that optimality is not ruled out, even if one player (namely, P) is completely ignoring his own private information. It is obtained only when there is a high compensation from R's and I's sides. This means that they both must play the same strategy and this strategy tries to offset P's strong bias. It is relatively easy to understand why this result is correct. Assume that the true state is H: then no information is lost by P. In particular, his action in this case (but not his strategy) recalls that of the player who always votes "yes" when uninformed. To that action, I's best response is exactly the strategy we outlined here and, from a social welfare point of view, R can play any q. This profile has been already shown to be one of the optimal ones in section 3.3.1. Suppose now the true state is L: then the right decision is always taken! If R and I are informed, then they take the correct decision by definition. But if they are not, then they both play "no", which is exactly the correct choice to take.

Recalling the example we provided in subsection 3.3.1, we can show the similarities between this game and the one with three truth-seeker players. In the latter case, the committee was possibly wrong in two cases: conditional on H being the true state, either no player was informed or only j was informed. But this is exactly what happens here, except that we have player P instead of player j. So, for both the equilibrium strategy profiles, the probability that the final decision is wrong is the same.

Despite the multiplicity of suboptimal equilibria, we know that there is a unique socially optimal equilibrium. In particular, when both P and R play a dominant strategy, I's best response is unique. Multiplicity only arises due to the fact that R can choose any level of $q \in [0, 1]$.

Introducing heterogeneity is therefore relevant for the characterization of the equilibria of the model. Even though equilibria are still multiple, their range is dramatically reduced.

The sequential voting game

In the sequential voting game, the order of vote turns out to be relevant. We link this result to the presence of heterogeneous players as, so far, no optimal order has ever emerged. In the Proposition 11, we present the main result of the game and then we illustrate it.

Proposition 11 *The sequential voting game with three heterogeneous players has a unique* *socially optimal equilibrium when I votes before R. For other sequences, there are multiple equilibria, only one of which is socially optimal.*

To prove Proposition 11, we assume P to be a reformist. His action is totally uninformative, so his position is irrelevant. His strategy is:

$$s_P^* : \{\text{always play "yes"}\}$$

If R votes before I, he has no additional information on which to base his action, so he behaves as in the simultaneous voting game and therefore his best strategy is still:

$$s_R^* : \left\{ \begin{array}{c} \text{play } \omega_R \text{ if any;} \\ \\ \text{play "yes" with any probability } q \in [0,1] \text{ otherwise} \end{array} \right\}$$

As we know, the best I can reply at this point is to (partially) compensate P's behaviour and so he plays:

$$s_I^* : \left\{ \begin{array}{c} \text{play } \omega_I \text{ if any;} \\ \\ \text{play "yes" with probability } p = 0 \text{ otherwise} \end{array} \right\},$$

as in the simultaneous voting game of the previous subsection.

We have already shown in the previous subsection that the first best is only achieved when $q = 0$. In general, there is a continuum of equilibria which are not optimal.

On the contrary, when I votes first, he still finds it optimal to play his usual strategy:

$$s_I^* : \left\{ \begin{array}{c} \text{play } \omega_I \text{ if any;} \\ \\ \text{play "yes" with probability } p = 0 \text{ otherwise} \end{array} \right\}$$

whereas R can now elicit some information from I's behaviour and so he plays:

$$s_R^* : \left\{ \begin{array}{c} \text{play } \omega_R \text{ if any;} \\ \\ \text{follow } I \text{ if } \Omega_R = \{H, L; d_I\} \end{array} \right\}$$

This strategy is optimal for R: as there is a positive probability that I is informed and therefore correct. But this strategy is socially optimal as well, because, when I is informed, then an uninformed R imitates the correct choice. And if I is uninformed as well, then he plays "yes" with probability $p = 0$ and R does the same, playing "yes" with probability $q = 0$. This is exactly the optimal equilibrium profile we found in the simultaneous voting game. Moreover, this equilibrium is clearly unique. So the sequential structure actually works as an "implementation mechanism" for the optimal strategy profile.

3.6 Possible extensions of the basic models

In this section we informally discuss two natural possible extensions of the basic models introduced in section 3.3 and 3.4. The first one regards the accuracy of the signal (the parameter β), while the second one concerns the number of players.

We refer again to the games as presented in Table 3.2, which we reproduce below for simplicity.

Table 3.2 (revisited)

	Truth	Reputation
Simultaneous	Game A	Game C
Sequential	Game B	Game D

3.6.1 The effect of noisy information

We now assume $\beta \in (\frac{1}{2}, 1]$: the signal observed by the players is not necessarily correct. As we are dealing with "experts", we find it reasonable to assume that the common β should be greater than $\frac{1}{2}$. This choice is also without loss of generality, as, if $\beta < \frac{1}{2}$, then the utility-maximizing behaviour would be just the opposite. With a positive probability of wrong signals, we expect herding to arise, at least in the reputational sequential voting game.

The main result of the subsection is that noisy signals either do not influence the players' equilibrium strategies or, if they do, they are not a source of inefficiency[16].

Game A: simultaneous voting with truth purpose Consider the usual strategy for player $i(j,z)$:

$$s_{i(j,z)} : \left\{ \begin{array}{c} \text{play } \omega_{i(j,z)} \text{ if } \Omega_{i(j,z)} = \{\omega_{i(j,z)}\}; \\ \text{play "yes" with probability } p(q,r) \text{ if } \Omega_{i(j,z)} = \{H, L\} \end{array} \right\}$$

The first part of the strategy is optimal as a signal, despite its noisiness, unambiguously shifts the player's prior (remember that $\beta > \frac{1}{2}$). We then solve for the utility maximizing p, q and r as before. We obtain that these probabilities are independent on the parameters

[16] We now present only some intuitions, leaving more formal proofs to future research.

α and β. Therefore equilibrium strategies are unaffected. These two parameters influence only the expected value of election, which is indeed lower for lower values of α and β.

Before analyzing game B, we focus on games with reputational purpose, as some of the results will be useful for the remaining case.

Game C: simultaneous voting with reputational purpose We already know that, in this simultaneous game, a player's optimal strategy is independent from the others' and therefore it is a dominant strategy. Consider the generic player i, playing the following strategy:

$$s_i : \left\{ \begin{array}{c} \text{play } \omega_i \text{ if } \Omega_i = \{\omega_i\}; \\ \text{play "yes" with probability } p \text{ if } \Omega_i = \{H, L\} \end{array} \right\}$$

Again, the first part of the strategy is optimal, as a signal shifts the player's prior (equal to $\frac{1}{2}$) about the true state of the world. We focus on p. It is straightforward to show that, again, p is independent of α and β. In particular, the expected utility of the election is given by:

$$E(u_i) = \frac{1 + 2\alpha\beta - \alpha}{2}$$

which is non negative for any $\alpha \in [\frac{1}{2}, 1]$ and $\beta \in (\frac{1}{2}, 1]$.

Game D: sequential voting with reputational purpose We begin with a definition, which clarifies what we mean by "following" and "herding" in the rest of the section. We recall that, with reputational purposes, the best choice is the one which is most likely to be correct.

Definition 4 (Herding and imitiation) *We say that the generic player "j" is "imitat-*

ing" the generic player "i" when his signal is consistent with i's behaviour or when j is uninformed. In line with the literature, we say that "j" is "herding" when he disregards his own private information.

We must stress that a player always makes the best use of his available information. If he does not have any signal, then he finds it optimal to imitate someone who might have observed some signal. If he does have a signal, then he updates his prior according to all his information and decides whether to follow his signal or herd.

For simplicity, we assume that the three players are voting in this order: i, j, z. Simply by using Bayes' rule, it is easy to show that herding might arise only from the third player (z) onwards. In particular, herding requires four main conditions. First of all, the first two players (i and j) must vote for the same option. Secondly, z must be informed. Otherwise, he is not giving up any information and simply imitating (from Definition 4). Then, his signal must contrast with the choice of i and j. Finally, the following Condition about p, α and β must hold[17]:

Condition 1 *Herding condition:*

$$p < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta),$$

where α and β are known parameters, respectively about the probability of observing a signal and its accuracy, and p is the probability that, when uninformed, i plays "yes".

[17] This condition is true when i and j vote "yes" and z observes the L signal. See the next footnote for the other case.

Condition 1 means that when α is very low ($\alpha \simeq \frac{1}{2}$), it is unlikely that both i and j observe a signal: for many values of the probability p, z trusts his signal and does not herd. When α grows, it's the term β which makes the difference. The higher the β, the lower the p sustaining herding: z has stronger confidence in his own signal. Finally, when $\alpha = 1$, herding is a straightforward conclusion for any β and p[18].

The conclusion of this discussion is that, in this case (with three players), herding is not inefficient from a social point of view, as it emerges only when the final decision has already been taken (z is not pivotal).

Game B: sequential voting with truth purpose This game mixes characteristics of the previous sequential game and of game A. So, we can easily conclude that, if herding arises, it is from the third player and it is therefore inefficient.

Comments

We start from a comparison between the general case when $\beta \in (\frac{1}{2}, 1]$ and the special one when $\beta = 1$. We can observe that nothing changes for players with truth purpose in a simultaneous game. The expected value is lower for smaller β, but the equilibrium strategies are the same. When players are status seekers, the situation is more similar to a typical model of herd behaviour. For different values of α, β and p, herding may arise. The third player finds it profitable to give up his private information about the true state of the world when the probability that other players observed a signal is very high and the accuracy of his information is not very sharp.

[18] In the symmetric case, where both i and j vote "no" but z has an H signal, the herding condition is $(1-p) < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta)$.

Finally, the possibility of maximizing social welfare is not dramatically reduced when we relax our assumption about β. This is clearly true for games A, B and C, as the equilibrium strategies do not change. In game D, herding may arise and it is inefficient from a social point of view. In this particular case, though, a committee with three members can not really do worse than in the previous case ($\beta = 1$): herding only arises when the voter is no longer pivotal (he cannot influence the final outcome of the game).

3.6.2 Changing the number of players

A further natural extension of our models is the analysis of committees composed of a different number of members.

First of all, we note that in game C the equilibrium strategy of each of the players is independent from the others, so the number of players is not an issue. We provide some intuitions for the other cases, when $n = 2, 4, 5$, but leave their analytical development to future research.

Game A: simultaneous voting with truth purpose When $n = 2$ or, in general, with an even number of voters, we need a tie-breaking assumption. We assume that the alternative passes when it receives $\frac{n}{2} + 1$ votes; in other words, the *status quo* wins in case of parity. Given two players i and j, it is easy to work out their equilibrium strategy. Following our previous notation, we call p and q respectively the probability that i and j play "yes" when uninformed. Then the optimal probabilities are $p = q = 1$. Compensation between players does not emerge when $n = 2$. But we must stress the existence of a strong bias, that is, the tie-breaking rule favouring the *status quo*[19].

[19] With an opposite tie-breking rule, we obtain $p = q = 0$.

When $n = 5$, the optimal strategy requires four members compensating each other when uninformed, with the last one free to play “yes” with any probability. Unfortunately, we do not have a formal proof for this claim. Furthermore, the expected value of the election is higher with 5 players than with 3 when they play the (putative) optimal strategy. This result is in line with previous research stating that the more players, the finer the aggregation of information.

Game B: sequential voting with truth purpose When $n = 2$, it is very easy to work out the optimal equilibrium strategy. Actually, it fully replicates game A. Recall from above the definition of p, q_Y and q_N: with the tie-breaking rule we chose, q_N is irrelevant as a decision has already been taken and we reduce the problem to the one we have just solved above: $p = q_Y = 1$.

As the number of players grows, calculations become more and more difficult. We think the basic result of the 3 players case still holds, that is, the sequential structure cannot improve the expected value of the election.

Game D: sequential voting with reputational purpose We know that for $n = 3$, herding may arise when the first two members (i, j) voted the same and the third player (z) has a signal in contrast with their choice. In particular, z herds when Condition 1 is satisfied. But from a social point of view, herding is not a problem, as z is never pivotal when it occurs.

When $n = 4$ (and, *a fortiori*, for higher values of n), the player after z (say, k) may face a similar situation: he observes i, j, z voting the same but has an opposite signal. Then, he should herd as well. The intuition is that, if Condition 1 is enough for a cascade to start after two players, then it must be the case that it is enough to start after three. And we can show that herding always arises (or continues). On the contrary, when k is informed but does not observe three similar votes, he knows that he is not the only one with that signal and should therefore consider it and play it.

With $n > 4$ players, herding might become dangerous from a social welfare point of view, because pivotal members of the committee are ignoring their own information.

3.7 Conclusions

As highlighted in the introduction, this chapter tries to answer to two main questions. The first one is about the voting behaviour of socially oriented voters, when they are uninformed and cannot abstain. The second is about the rationale for the presence of biased members in committees whose aim is to maximize social welfare.

To do this, we solve and comment on different voting games. We first discriminate among games played sequentially and games played simultaneously. Some papers have already discussed how, in the former case, the probability of herding may arise and the information aggregation process of the election may be inefficient. Indeed, by herding, players give up their private information and concur to create informational cascades.

We analyze the particular case of a committee composed of 3 members, which must take a binary decision. For instance, it may decide whether to accept an alternative or reject it and keep the status quo (which has a normalized value of 0). In case of acceptance, the alternative can take two values: either 1 or -1, according to the true state of the world. Abstention is not possible.

When players do not want to share their private information, the information aggregation process of the voting game may be flawed. This is not necessarily true in a setting with simultaneous voting and a wide number of players but can be a serious consequence otherwise.

The traditional conclusions of the existing literature about sequential voting appear to be strongly dependent on the objective function of the players. For this reason, we also discriminate according to the members' objective functions. We show that herding still arises if the committee is composed of players with reputation objectives (they want to maximize their own probability of being right). But this is no longer true when players have a truth purpose, that is, when they want to maximize social welfare (defined as the probability that the committee takes the correct decision). They might in fact decide to destroy possible informational cascades, playing compensating strategies instead of herding. In particular, optimality requires compensation in pure strategies. Compensation is an optimal behaviour if players cannot abstain. Nevertheless, it prevents full aggregation of available information and for this reason the optimality concept we refer to is defined as "constrained" one.

The second main contribution of the chapter consists of the analysis of a committee with heterogeneous members. Quite surprisingly, in the existing literature heterogeneity has exclusively regarded the ability of the players (i.e., the noisiness of their signals/private information). In our model, players with the same ability have in fact different preferences. We model our environment on the peculiar composition of the Italian Constitutional Court. According to the Italian Constitution, one-third of its members are appointed by the Parliament, one-third by the President of the Republic and one third by the highest judges. The historical and political reasons for that pertain to the desire for a balance among political and jurisdictional power. We argue whether this composition is also efficient in terms of social welfare (probability of taking the right decision). Accordingly, we assume the existence of three members: a truth-seeking member, a status-seeking member (with reputation purpose) and a politically biased member. The answer is very surprising: even with a politically biased player, who constantly ignores his private information and favours his political party, there exists an equilibrium that is socially optimal. In the simultaneous game, optimality requires some coordination, which may not necessarily be realized. On the contrary, in the sequential game both optimality and uniqueness are reached, given a particular order of voting. The idea is that the dominating strategy of the biased member destroys the uncertainty about his behaviour, leaving the social maximizer able to react in a socially optimal way. By constraining the status seeker to vote after the social maximizer, the former is able to extract useful information from the latter's decision, thus realizing the necessary coordination required by the social optimal equilibrium.

3.8 Appendix: proofs

3.8.1 Proof of Proposition 6

We provide a proof for Proposition 6, which nests the proof for Proposition 5 as well.

Consider the following strategy:

$$s_{i(j,z)} : \left\{ \begin{array}{c} \text{play } \omega_{i(j,z)} \text{ if } \Omega_{i(j,z)} = \{\omega_{i(j,z)}\}; \\ \text{play "yes" with probability } p(q,r) \text{ if } \Omega_{i(j,z)} = \{H, L\} \end{array} \right\},$$

The first part of the strategy must be optimal, as the signal is always correct. For the second, we want to solve for the profit maximizing probabilities. When the actual value of the alternative is $v_m = 1$ (*ex ante* possible with probability equal to $\frac{1}{2}$), i plays "yes" with probability $\alpha + (1-\alpha)p$ (similarly for j and z); this is the probability that he has the correct signal (H) plus the probability of playing "yes" when he has no signal. We define the probability for the committee as a whole to vote "yes" to be a function $Y(p,q,r)$, where the probability of voting "no" is then $1-Y(p,q,r)$. The expected value of the election when the actual value of the alternative is 1 is then given by:

$$\begin{aligned} E(v_m|v_m &= 1) = 1\left[Y(p,q,r|v_m=1)\right] + 0\left[1 - Y(p,q,r|v_m=1)\right] \\ &= Y(p,q,r|v_m=1) \end{aligned}$$

The alternative passes when it receives at least two votes. The probability that all the members vote "yes" is:

$$[\alpha + (1-\alpha)p]\,[\alpha + (1-\alpha)q]\,[\alpha + (1-\alpha)r] \tag{3.1}$$

The probability that only two members vote "yes" is:

$$\begin{aligned} &[\alpha + (1-\alpha)p]\,[\alpha + (1-\alpha)q]\,(1-r)(1-\alpha) + \\ &[\alpha + (1-\alpha)p]\,(1-q)(1-\alpha)\,[\alpha + (1-\alpha)r] + \\ &(1-p)(1-\alpha)\,[\alpha + (1-\alpha)q]\,[\alpha + (1-\alpha)r] \end{aligned} \tag{3.2}$$

Therefore:

$$E(v_m|v_m = 1) = Y(p,q,r|v_m = 1) = (3.1) + (3.2)$$

When the actual value of the alternative is $v_m = -1$ (ex ante possible with probability equal to $\frac{1}{2}$), i plays "yes" with probability $(1-\alpha)p$ (similarly for j and z), as the only possible signal is L. The expected value of the election is given by:

$$\begin{aligned} E(v_m|v_m &= -1) = -1\,[Y(p,q,r|v_m=-1)] + 0\,[1 - Y(p,q,r|v_m=-1)] \\ &= -Y(p,q,r|v_m=-1) \end{aligned}$$

The probability that all the members vote "yes" is:

$$(1-\alpha)^3 pqr \tag{3.3}$$

The probability that only two members vote "yes" is:

$$\begin{aligned} &(1-\alpha)^2 pq\,[\alpha + (1-r)(1-\alpha)] + \\ &(1-\alpha)^2 pr\,[\alpha + (1-q)(1-\alpha)] + \\ &(1-\alpha)^2 qr\,[\alpha + (1-p)(1-\alpha)] \end{aligned} \tag{3.4}$$

Therefore:

$$E(v_m|v_m = -1) = -Y(p,q,r|v_m = -1) = (3.3) + (3.4)$$

Finally, the expected value of the game is:

$$E(v_m) = \frac{1}{2}[Y(p,q,r|v_m = 1) - Y(p,q,r|v_m = -1)]$$

The equilibrium levels of p, q and r are obtained from the following:

$$\max_{p,q,r} E(v_m) = E(p, q, r)$$

The (simplified) first order conditions for this problem are independent of α and given by:

$$\begin{cases} \frac{\partial E(v_m)}{\partial p} : 1 - q - r = 0 \\ \frac{\partial E(v_m)}{\partial q} : 1 - p - r = 0 \\ \frac{\partial E(v_m)}{\partial r} : 1 - p - q = 0 \end{cases}$$

with the following solutions:

$$p = \begin{cases} 1 & \text{if } 1 - q - r > 0 \\ 0 & \text{if } 1 - q - r < 0 \\ \in [0, 1] & \text{if } 1 - q - r = 0 \end{cases};$$

$$q = \begin{cases} 1 & \text{if } 1 - p - r > 0 \\ 0 & \text{if } 1 - p - r < 0 \\ \in [0, 1] & \text{if } 1 - p - r = 0 \end{cases};$$

$$r = \begin{cases} 1 & \text{if } 1 - q - p > 0 \\ 0 & \text{if } 1 - q - p < 0 \\ \in [0, 1] & \text{if } 1 - q - p = 0 \end{cases}.$$

The only interior solution is $\left\{r = q = p = \frac{1}{2}\right\}$, which is the symmetric equilibrium strategy profile.

In addition, we have the following corner solutions:

$$\begin{aligned}\{p,q,r &\in [0,1]^3 : q+r=1, qr=0\},\\ \{p,q,r &\in [0,1]^3 : p+q=1, pq=0\},\\ \{p,q,r &\in [0,1]^3 : p+r=1, pr=0\}.\end{aligned}$$

3.8.2 Proof of Proposition 7

Given the following order of vote: i, j, z, we characterize the players' strategies as follows:

$$s_i : \left\{\begin{array}{c}\text{play } \omega_i \text{ if } \Omega_i = \{\omega_i\};\\ \text{play "yes" with probability } p \text{ if } \Omega_i = \{H, L\}\end{array}\right\},$$

$$s_j : \left\{\begin{array}{c}\text{play } \omega_j \text{ if } \Omega_j = \{\omega_j, d_i\};\\ \text{if } \Omega_j = \{H, L; d_i\}\text{:}\\ \text{play "yes" with probability } q_Y \text{ when } d_i = \text{"yes"}\\ \text{play "yes" with probability } q_N \text{ when } d_i = \text{"no"}\end{array}\right\},$$

$$s_z : \left\{\begin{array}{c}\text{play } \omega_z \text{ if } \Omega_z = \{\omega_z, d_i, d_j\}\\ \text{if } \Omega_z = \{H, L; d_i, d_j\}\text{:}\\ \text{play "yes" with probability } r_{YY} \text{ when } d_i = d_j = \text{"yes"}\\ \text{play "yes" with probability } r_{YN} \text{ when } d_i = \text{"yes" and } d_j = \text{"no"}\\ \text{play "yes" with probability } r_{NY} \text{ when } d_i = \text{"no" and } d_j = \text{"yes"}\\ \text{play "yes" with probability } r_{NN} \text{ when } d_i = d_j = \text{"no"}\end{array}\right\}$$

Recalling the definition of function $Y(\cdot)$, the problem we face is the following:

$$\max_{p,q_N,q_Y,r_{NY},r_{YN}} E(v_m) = \frac{1}{2}\left[Y(p,q_N,q_Y,r_{NY},r_{YN} \mid v_m = 1) - Y(p,q_N,q_Y,r_{NY},r_{YN} \mid v_m = -1)\right]$$

First of all, we notice that if we fix $q_Y = q_N = q$ and $r_{YN} = r_{NY} = r$ (i.e., the players do not discriminate), then we go back to the previous case (simultaneous game) where the players' actions were not interdependent. So, every equilibrium profile in the simultaneous voting game is still an equilibrium in the sequential one. We avoid writing specific formulas for these functions, as they are straightforward extensions of the previous case. We focus on the first order conditions (after useful simplifications) in order to understand what the range of possible equilibria is:

$$\begin{cases} \frac{\partial E(v_m)}{\partial p} : 2 - q_N - q_Y - r_{NY} - r_{YN} = 0 \\ \frac{\partial E(v_m)}{\partial q_N} : 1 - p - r_{NY} = 0 \\ \frac{\partial E(v_m)}{\partial q_Y} : 1 - p - r_{YN} = 0 \\ \frac{\partial E(v_m)}{\partial r_{NY}} : 1 - p - q_N = 0 \\ \frac{\partial E(v_m)}{\partial r_{YN}} : 1 - p - q_Y = 0 \end{cases}$$

As expected, the symmetric solution is still an equilibrium profile:

$$\left\{ r_{YN} = r_{NY} = q_Y = q_N = p = \frac{1}{2} \right\}.$$

This is still the only interior solution.

In addition, there are the following corner solutions:

$$\left\{ \begin{array}{c} r_{YN} = r_{NY} = 1 - q_Y = 1 - q_N; \\ \\ q_N r_{YN} = 0; \forall p \in [0,1] \end{array} \right\},$$

which is an equilibrium in the simultaneous game as well; and

$$\{r_{YN} = r_{NY} = 1 - p; pr_{YN} = 0; \forall q_Y, q_N \in [0,1]\}$$

$$\{q_N = q_Y = 1 - p; pq_Y = 0; \forall r_{YN}, r_{NY} \in [0,1]\}$$

which is a straightforward extension of the previous solution: once two players compensate each other, it does not make any difference whether the third is discriminating (as in the sequential game) or not (simultaneous game). Finally, we have the most different profile, where both the player differentiate their actions:

$$\left\{ \begin{array}{c} r_{YN} = 1 - q_Y \neq r_{NY} = 1 - q_N; \\ \\ r_{YN} q_Y = 0; r_{NY} q_N = 0; \\ \\ \forall p \in [0,1] \end{array} \right\}$$

3.8.3 Proof of Condition 1

In the sequential voting game, the first player (say, i) is in the same situation as any generic player in game C. When informed, he plays the signal; when uninformed, he plays "yes" with any probability $p \in [0,1]$. As we show in the chapter, he is correct with *ex ante* probability $\frac{1+2\alpha\beta-\alpha}{2}$. As regards following players, they can elicit some information from previous voters' actions. We use Bayes' rule to update the priors and we define the following events:

$$Y \quad : \quad \text{the true state is "Yes";}$$

$$X \quad : \quad \text{the previous player(s) voted "Yes"}$$

$$\text{and the current player has a "No" signal.}$$

Then, we introduce the herding condition:

$$\Pr[Y \mid X] > \frac{1}{2} \tag{3.5}$$

where, by definition:

$$\Pr[Y \mid X] = \frac{\Pr\left[Y \cap X\right]}{\Pr[X]} = \frac{\Pr[Y] \Pr[X \mid Y]}{\Pr[Y] \Pr[X \mid Y] + \Pr[\bar{Y}] \Pr[X \mid \bar{Y}]}$$

The herding condition in (3.5) states that, if the posterior belief (left hand side) is greater then the indifference point (right hand side), then the current player should ignore his own private information and follow the previous player(s).

Starting from the second player (say, j), he faces three possible cases:

- – If he is uninformed, he should follow i, because he is right with probability $\frac{1+2\alpha\beta-\alpha}{2} \geq \frac{1}{2}$;
  - If he is informed and the signal is in line with i's choice, then there is a higher probability that the signal is correct. So, j should play his signal (or follow i, which is equivalent).
  - If he is informed but the signal contrasts with i's choice, j prefers to follow his

signal. We do not need a tie breaking assumption (as, for instance, Banerjee, 1992). From our condition in (3.5), we obtain:

$$\frac{\frac{1}{2}\left[\alpha\beta+p(1-\alpha)\right]\alpha(1-\beta)}{\frac{1}{2}\left[\alpha\beta+p(1-\alpha)\right]\alpha(1-\beta)+\frac{1}{2}\left[\alpha(1-\beta)+p(1-\alpha)\right]\alpha\beta}>\frac{1}{2}$$

which is never true. This means that j should always play according to his own signal. He knows that i could indeed be uninformed, whereas he knows he has a signal and should put more weight on it than on i's choice.

Finally, the third player (say, z), faces the following possible cases:

- - If i and j played the same and z has no signal, then he should follow.
  - If i and j played the same and he has a consistent signal, the probability that they are all right is certainly higher than $\frac{1}{2}$. Then he still should follow (or, equivalently, play his signal).
  - If i and j played the same and he has a different signal, then z may herd. This is the new result of the subsection, which is a direct consequence of introducing the noisiness of the signal. Suppose both i and j play "yes" but z has an L signal, indicating "no" as the correct choice. Recalling that i plays "yes" with probability p when uninformed, (3.5) becomes:

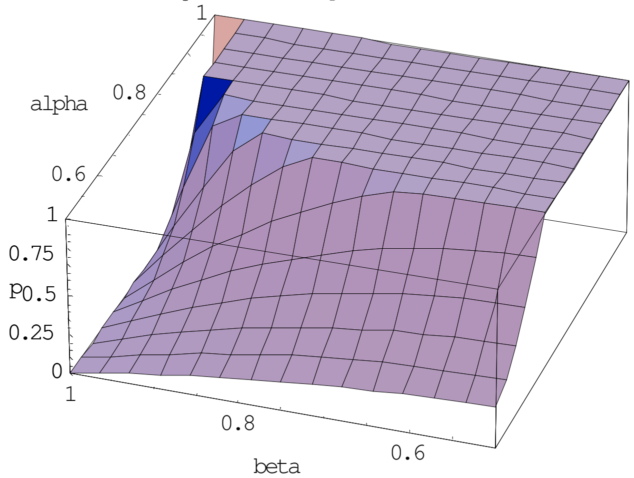
$$\frac{\frac{1}{2}[\alpha\beta+p(1-\alpha)][\alpha\beta+1-\alpha]\alpha(1-\beta)}{\frac{1}{2}[\alpha\beta+p(1-\alpha)][\alpha\beta+1-\alpha]\alpha(1-\beta)+\frac{1}{2}[\alpha(1-\beta)+p(1-\alpha)][\alpha(1-\beta)+1-\alpha]\alpha\beta}>\frac{1}{2}$$

in which it is important to note that, if uninformed, j always follows i. The condition is true if and only if

$$p < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta) \tag{3.6}$$

So, according to different values of p, herding may arise or not. Graph 3.2 presents a tridimensional plot of the function $\left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta)$:

Graph 3.2: Herding condition for z



When α is very low, it is unlikely that i and j observed a signal: for high values of the probability p, z prefers to trust his signal and does not herd. When α grows, it is the term β which makes the difference. The higher β, the lower p which sustains herding: z has stronger confidence in his own signal. Finally, when $\alpha = 1$, herding is a straightforward conclusion for any β and p, a part from the case when $\beta = 1$: z fully relies on his signal and never herd[20].

– If i and j played the opposite, this means that j was informed. If z is uninformed,

[20] We recall that, when both i and j vote "no" but z has an H signal, the herding condition is $1-p < \left(\frac{\alpha}{1-\alpha}\right)^2 \beta(1-\beta)$. This is obtained with a simple modification of the condition in (3.5).

then he should follow j, as $\beta \geq \frac{1}{2}$. If z has a consistent signal, then he should follow. Finally, if his signal his in line with i's choice, then he plays his signal. Again, this last result is obtained through Bayes' rule. The intuition is simply that z's signal is slightly reinforced by the possibility that i is informed as well and that j's signal is wrong. To prove this, we start by defining the following events:

$$\begin{aligned} Y &: \text{ the true state is "Yes";} \\ W &: \text{ the first player voted "No", the second "Yes"} \\ &\quad \text{ and the current player has a "No" signal.} \end{aligned}$$

Then, the herding condition requires:

$$\Pr[Y \mid W] > \frac{1}{2}$$

that is:

$$\frac{\frac{1}{2}[\alpha(1-\beta)+(1-p)(1-\alpha)]\alpha^2\beta(1-\beta)}{\frac{1}{2}[\alpha(1-\beta)+(1-p)(1-\alpha)]\alpha^2\beta(1-\beta)+\frac{1}{2}[\alpha(1-\beta)+(1-p)(1-\alpha)]\alpha^2\beta(1-\beta)} > \frac{1}{2}$$

which is never true.

3.8.4 Full information aggregation

We can imagine this situation as one where a single player (the *decision maker*) collects all the available information from the others and then takes a decision. It is straightforward to understand that the correct decision is always taken when at least one of the members has a signal, as this is always correct. When nobody is informed, the decision maker votes "yes" with any probability $p \in [0,1]$, which is optimal, given the priors about the true state of the world.

The expected value of the election when the actual value is 1 is given by:

$$\begin{aligned} E(v_m|v_m &= 1) = 1\left[Y(p,q,r|v_m=1)\right] + 0\left[1 - Y(p,q,r|v_m=1)\right] \\ &= Y(p,q,r|v_m=1) = \alpha^3 + 3\alpha^2(1-\alpha) + 3\alpha(1-\alpha)^2 + p\,(1-\alpha)^3 \end{aligned}$$

The expected value when the actual value is -1 is given by:

$$\begin{aligned} E(v_m|v_m &= -1) = -1\left[Y(p,q,r|v_m=-1)\right] + 0\left[1 - Y(p,q,r|v_m=-1)\right] \\ &= -p\,(1-\alpha)^3 \end{aligned}$$

Therefore:

$$\begin{aligned} E(v_m) &= \frac{1}{2}[Y(p,q,r|v_m=1) - Y(p,q,r|v_m=-1)] \\ &= \frac{1}{2}\left[\alpha^3 + 3\alpha(1-\alpha)\right] \in \left[\frac{7}{16}, \frac{1}{2}\right] \text{ for } \alpha \in \left[\frac{1}{2}, 1\right] \end{aligned}$$

Part II

The Use of Power

Chapter 4

Models of Partnerships

4.1 Introduction

The debate about public and private provision of public goods and services has always been lively, both in the political and in the academic arena. The application of the "incomplete contracts" framework has enriched this debate. The importance of contractual relationships is now even more emphasized by the increasing relevance assumed by partnerships between public (central or local) authorities and private firms. In this chapter, we refer in particular to Public-Private Partnerships (henceforth, PPPs). The aim of this work is to present the main theoretical contributions to this debate, to discuss some experiences across UK[1], to introduce our models of partnerships and workers' contribution to performance and, finally, to provide policy suggestions.

The introduction of workers' incentives is our main contribution. We believe that political debate has said probably too much and conversely academia too little about this topic.

[1] An excellent review about some Italian cases can be found in Ambrosanio, Bordignon and Etro (2004).

As a matter of fact, most of the political debate is nowadays focusing exactly on this point: who should employ whom. For example, recent guidances for new partnership contracts in Scotland[2] explicitly underline that workers and unions should be informed about the privatization process, that workers' conditions are essential for a satisfactory provision of the service and that two tier workforce should be limited. By two tier workforce, we refer to the coexistence of workers who are employed under different conditions. This is indeed a very common situation in partnerships, where some of the staff is usually transferred from a public to a private employer.

The chapter is organized as follows. In section two, we define partnership contracts, discuss some of the main UK experiences of partnerships, and highlight advantages and drawbacks of these choices. In section three, we present a review of the main theoretical literature. In section four we introduce our main contribution: a model allowing for public provision, beside private provision and PPP, and the presence of workers. We divide the section in two parts, in order to have simplified versions of our model and better focus our analysis. Finally, in section five we draw our general conclusions and provide policy suggestions implied by the model[3].

4.2 Partnerships in UK

Partnerships are not a completely new device for the delivery of public services. For instance, Scott Fosler and Berger (1982) witness the presence of partnerships in seven U.S. municipalities during the '70s. More recently, Rossenau (2000) reports evidences from

[2] *PFI Quarterly*, various numbers. Available at http://www.scotland.gov.uk/pfi.

[3] I thank Jozsef Sakovics, Jonathan Thomas and John H. Moore for their suggestions and supervision. I also thank the participants in the 2005 SGPE Conference in Dunblane (Scotland) for their comments. I am especially grateful to Piergiovanna Natale and Santiago Sanchez-Pages for valuable suggestions.

successful American and British experiences. Nevertheless, the history of partnerships is not always a list of successes: some of the first partnership contracts failed to be satisfied and still nowadays criticism arises when a new partnership is proposed[4].

The rest of the section is organized as follows: first we draw a brief historical background of the policies which brought to the introduction and development of PPP contracts in UK; then, we analyze different types of partnerships and try to understand the main elements of these contracts. Finally, we present a taste of UK experience and, in particular, we focus on health, education and prisons.

4.2.1 From traditional private provision to PPP

The main arguments supporting the shift from public to private provision are usually connected to the necessity of solving budget problems, without raising extra revenues, and the desire to obtain efficiency gains.

The first wave of privatization in the UK spread with PM Margaret Thatcher, during the eighties. Despite some expected benefits, new problems emerged: the government sold most of its assets at an excessively low price, competition was not always possible, and benefits were unevenly distributed among management and employees (HM Treasury, 2000).

The first half of the last decade opened with the introduction of Private Finance Initiative (henceforth, PFI) contracts: in this way, the government wished to keep the level of public investments high, to provide further incentives for private capital and, at the same time, to retain an overall public control on the projects. Pollit (2000) classifies three kinds

[4] See, for example, www.unison.co.uk for the Scottish case.

of PFI projects:

1. the public sector buys the service (e.g.: roads, prisons) from the private sector, which is responsible for the capital investments;

2. the private sector designs, builds, finances and operates an asset; fees are paid by the public authority over the life of the contract, providing the required standards are met (Bennett and Iossa, 2004, and Hart, 2003, are based on these particular contracts, henceforth referred as DBFO); according to the nature of the asset, the private sector might in some cases directly charge the users (e.g., a bridge);

3. joint ventures between private and public sector.

Research, debate and propaganda have mainly focused on the following most relevant elements characterizing PFI contracts:

- the possibility of delaying payments as long as the contract would last;
- the transfer of risks from the public to the private sector;
- the capacity to provide "value for money": a PFI should be set up only if the output could be provided at a cheaper cost than by a different type of provision (on a cost-benefits analysis).

As it will become clearer later, we believe in Hart's view, that is that PFIs find their peculiarity more in the length of the contract involved rather than the listed points. We add to his theory the role of workers' incentives which, at least initially, was not even considered by policy makers.

These main characteristics (especially risk transfer and value-for-money) were also the limits of this wave of PFI contracts. Even if the point is still debated (for example, see Grout, 1997), it is not clear why the cost of the private project should be lower, as borrowing for the private is more expensive than for the Government. Moreover, following Clark and Root (1999, p. 352), "risk was assumed to involve: design and construction risk (...); commissioning and operating risk (...); demand risk (...); residual value risk (...); technology and obsolescence risk". It is straightforward to understand that most of these risks could not be specified *a priori* for a lot of projects. Even lack of experience and of project management skills in the public sector were such that progress in PFI projects in the early years was very slow.

Despite all these limitations, first failures could provide lessons for future agreements. It is on the basis of these experiences that PPP contracts have been launched in Britain since 1997.

4.2.2 The elements of a PPP

Although they are often used as synonyms[5], PPPs encompass a wider variety of relationships than PFIs. According to Broadbent and Laughlin (2003), the main aspect differentiating PPPs from traditional private provision is the presence of some control from the public authority over the nature and pricing of the service offered. This control is possible through the exploitation of ownership rights. Now, as they recognize, this point is highly debatable. Even if assets are not always technically owned by the public, we believe there is still scope for economic (or political) ownership. In particular, it is important to

[5] This is true especially in academic research. According to this tradition, we decided to commit the same language abuse through the paper.

understand who is legally entitled to residual rights of control, which define what we call "economic ownership". We will soon come back to this important point when discussing the flexibility of contracts.

A first synthetic definition of partnership, obtained by collecting data from different sources (see, for example, HM Treasury, 2000, p. 10, and IPPR, 2001, p. 40), is "a long term risk sharing relationship between the public and the private sector to realize a mutual benefit". This definition is quite comprehensive of the main elements of a partnership, which are listed and more extensively commented on below.

- Private and public sectors. As regards the subjects involved, the definition above refers to private firms (for profit ones, voluntary sector and charities) and public authorities, being them central or local (but also, for example, hospitals trusts and central government departments). The main responsibilities for the latter subjects are political ones, that is: deciding and defining the objectives, fix the standards (above all, quality and quantity levels), monitoring the performances and ensuring that public interest is safeguarded. In some sectors, the public authority also provides part of the staff. The contribution of the private sector is supposedly based on its better management and business skills and efficiency driven culture. More precisely, public and private sectors are only words, which nest a lot of different stakeholders: customers, employees, private sector investors, banks and taxpayers. Most of the existing theoretical models often fail to consider the issues brought by each of these different agents. In this sense, our model acts as a first step, through the concern given to public and private sector employees. Taking into account that benefits must be shared among all the stakeholders is becoming only nowadays a fixed point in the political design of PPPs.

- Risk sharing. Within a partnership, risks should be borne by the party who can best manage them. Normally, political risk, plus a share of market risk, is retained by the public. It is this subject who is responsible for satisfying social needs. In addition, charges might be still due to the private firm even if the demand for the service drops (e.g., a demographic change leads to less pupils in a school). On the contrary, construction, design, standard satisfaction, operating costs and delivery risks are transferred to the private sector (and then reallocated within the consortium, when existing). There is also a final risk associated to the residual value of the facility, which is borne by the party who owns it.

- Long term. The duration of a contract may vary from five to seven years for local authorities outsourcing, and up to twenty-thirty years for schools. This "long-term" characteristic of the contracts and the size of the projects give rise to different problems. Pollit (2000) spots some particular difficulties. First, very often inputs and outputs cannot be specified in contracts, especially when they deal with quality issues. The less these items are sharply defined, the less the incentives for the private investors to respect the agreement. As already noticed, many risks are involved and transferred from the public authority to the private agent. In order to avoid problems (trials, delays, legal costs), they should be listed and specified. It is now very easy to understand the necessity to assume "incompleteness" in models dealing with this topic. Long-term contracts are also very hardly modifiable if unexpected contingencies realize during the provision period. This lack of flexibility is the source of well known hold up problems. In theoretical models, a no cost renegotiation could partially solve them. In reality, renegotiation is not always possible without breaking down the existing contract. With PFI contracts, key flexibility rights are given to the public sector (HM Treasury, 2003). In particular, provided an agreement on costs variation is found, the public sector has the right to change any aspect of the building or service provision. As anticipated above, this looks like a residual right of control over the asset, even if this is not necessarily owned by the public subject[6].

- Relationship. Contracts are usually tailored on the specific case following standardized schemes, in order to reduce writing and legal costs. Contracts specify outputs related to the service required by the public sector, rather then inputs specification and asset characteristics; the basis for payment is also an element of the contract. Outputs are typically designed in consultation with public sector workers (e.g., doctors and teachers). The public sector evaluates bids received from the private firms and selects an option. It is quite natural to expect a PPP to be signed when a public provision would turn out to be more expensive. Unfortunately, it is not easy to compare costs and benefits of two different providers. A PFI is likely to be chosen when it offers greater value for money, the investment horizon is sufficiently long (no less than five years) and outcomes can be well specified. Greater value for money means that the overall cost of the PFI satisfies a public sector comparator (PSC) criterion. This criterion involves comparisons about interest rates (the cost of borrowing), which are lower for a public subject, and tax on private profits (which can be transferred on the final purchaser of the good/service). The evaluation of the overall costs should be

[6] In other terms, if the public authority bears the cost of the proposed change (i.e., it undertakes the investment), then it does not need to share the additional benefit with the private firm.

made also with regard to other not economic aspects, such as employment condition. A further element of the contract is the performance evaluation and the system of deductions and penalties following poor standards levels. A constant monitoring of the performance of PFI projects is therefore required.

- Mutual benefit. Private and public sectors typically have different objectives. Private firms usually seek to maximize their profits whereas public authorities, at least in principle, wish to grant the highest benefit to the society. The difference might be less sharp when not-for-profit organizations are concerned but, again, they do not necessarily share the same utility function of society as a whole (on this point, see Dixit, 2002b).

Finally, following IPPR (2001, pp. 40-41) and HM Treasury (2000, pp. 46-48), we can list, and very briefly comment on, some possible types of PPPs:

1. PFI, as explained above, which constitutes the dominant form of partnership in UK. Variation of DBFO contracts are also possible, such as DBO or DBF agreements.

2. Wider markets: partnerships where private skills and finance should better exploit public assets or human resources.

3. Long term service provision contracts: these are agreements where no building stage is required but only management of existing assets.

4. Strategic (or policy) partnerships: agreements where the private sector is involved in the development and implementation of public services.

5. Sales of businesses: they involve the sale of shares of state-owned businesses, with

the hope that the presence of private investments and market discipline would release the full potential of these firms.

6. Joint ventures: partnerships with a pooling of public and private assets, finance and workers under a common management.

4.2.3 Some UK experiences

PFI in England plays a still limited but increasingly relevant role in public sector capital investment: 11% of total investment in public services in 2003-2004 is estimated to be due to PFI[7]. These investments have now delivered more than 600 new public facilities, including 34 hospitals, 119 other health schemes and 239 schools. PFI is used following a particular criterion, that is, it must offer value for money and efficiency gains must not be made at the cost of the workers' conditions. First evaluations seem encouraging: of 61 chosen projects, 89% were delivered on time and 77% of public sector managers were happy with the delivery. We now want to illustrate some particular experiences and cases of partnerships. There are many examples we could choose from, but we shall focus on schools, hospitals and prisons, as we find them particularly relevant and inspiring for the theoretical model we have in mind. We also concentrate on the employment aspects of these agreements. Other cases we do not have the space to discuss here, like London Underground, Post Office, National Air Traffic Service and British Nuclear Fuels, are exposed in Balduzzi (2000).

[7] If not differently and explicitly stated, the information in this subsection is based on HM Trasury (2000, 2003).

Health service and education

A common problem of these sectors (but probably of every PFI) was initially the difficulty to develop and write contracts. After the very first experiences, there is now a tendency to use standardized (but flexible) partnership contracts.

In the education sector, a wide typology of contracts is possible. For example (IPPR, 2001, p. 164):

- Design, build, finance and operate: this is the typical PFI contract, carried out by a consortium of private firms; this consortium owns the school over the entire period of the contract.

- Education Business Partnership: the private sector participates as future employer and community stakeholder.

- Dual use Facilities: the private sector can recover part of its costs by exploiting the facility for its own business.

- LEA (Local Education Authority) management and provision of services: the private sector provides only strategic services, such as management.

In any of the possible schemes (which can be mixed as well), the Head and the Governing Body of the school continue to be responsible for teaching, while cleaning and catering are provided by external staff. The idea is that this choice allows schools to focus on their main business (i.e.: education) and to raise their standards. Typically, some employees need to be transferred to the private contractor (e.g.: the school Caretaker, IT technicians) whereas the teaching staff will be unaffected. In this case, the transferred staff is protected by the TUPE (Transfer of Undertakings Protection of Employment) Regulation of 1981. Moreover, unions should be consulted as early as possible and service provider and Head teacher should develop close relationships between them.

In the health sector[8], PPPs always take the form of PFI contracts (in particular, DBFO). The consortium is usually required to build and maintain the facilities for the contract period. Employment design is similar to the one in education. The private sector provides ancillary services such as catering, cleaning, laundry security and portering. The public subject is responsible for the employment of clinical staff: it is believed that, in this way, the quality of the service would be better guaranteed. In addition, the contracts normally specify the range of services to be delivered, the performance standards required and the price of the provision, which is not due until services are provided to the agreed standard. The NHS has recently developed an original system called retention of employment (henceforth, RoE) in new PFI hospitals[9]. Under RoE, some categories of the ancillary staff are employed and retained as NHS employees but seconded to work for the consortium. The objective is to avoid a two-tier workforce but, in practice, the system is very complex and still highly debatable.

Prisons

Prisons constitute a really mixed sector. In England and Wales, custodial services are provided in 137 prisons (NAO, 2003), belonging to the public sector, the (traditional) private one or to PFIs. Among them, there are some interesting cases. Two prisons that were built and financed conventionally by the public sector are now run by private companies

[8] We focus here only on secondary care even if PPPs exist in primary and intermediate care as well.
[9] See http://www.ippr.org.

under management-only contracts. Three other prisons, two of which had previously been operated by the private sector, are now run by local management teams following successful in-house bids. Finally, since 1995, the Prison Service has signed nine Private Finance Initiative (PFI) contracts for new prisons, seven of which are already operational.

The most interesting aspect of this sector, at least according to the direction of our research, concerns employment choices. The full range of services, from management to staff, is here provided by the PFI. And it is a relevant chapter because staff costs account for nearly 80% of the running costs of a prison. It is natural to think that efficiency gains in the private sector might start exactly from the use of employees. And indeed the workforce has been subject to some reduction. Two main problems, related to staff conditions, are that staffing level in some prisons (e.g., HMP&YOI Ashfield; NAO, 2003) is failing to meet the original agreement. Furthermore, there is a high degree of turnover and a consequent lack of experience of constantly new employees. While most of the staff is recruited in the market or developed internally, senior managers positions are not. Directors have been recruited from the ranks of previous Prison Service Governors (public employees).

4.3 Partnerships, Contracts and Workers: a Review

Several contributions have recently discussed privatization and partnerships in an incomplete contracts framework. We also choose to develop our model in the well known "incomplete contracts" framework introduced in subsequent papers by Grossman and Hart (1986), Hart and Moore (1990) and finally Hart (1995). From an empirical point of view, it should be clear by now why partnership contracts would probably fail to be fully detailed. From a theoretical point of view, two main similarities about our models are worth of analysis. Among the other contributions they provide, these papers study the rationale for merging (or not) between private firms, whose objective is solely profit, and the optimal employment contract in these firms.

Hart, Shleifer and Vishny (1997) have already enlarged these models in order to consider a private firm and a government, whose objective function deals with quality rather than profit. This work provides criteria for the choice about contracting-out or not the provision of a public good, but it is not specific for PPPs. In a following paper (Hart, 2003), partnerships are explicitly considered. The author offers a new insight into the microeconomic principles that PPPs are (or should have been) based on. His preliminary conclusions are that conventional private provision (unbundling the construction of the facility and operation of the public service) is good if the quality of the building can be well specified in an initial contract, whereas the quality of the service cannot be. A series of short-term contracts (i.e.: more competition) will provide less distorted levels of investments. In contrast, PPP (bundling construction and operation) is good if the quality of the service can be well specified in a single long-term contract. Despite the advantages of his model, his analysis is still preliminary and fails to consider the effect of these two different provision devices on workers' efforts and the role of ownership.

First steps in the latter direction have been made by Besley and Ghatak (2001) and Bennett and Iossa (2004)[10].

The former contribution highlights how, when a public good is involved, ownership should be based on valuations and not on investments or technology. The authors argue that sometime it is socially better for private firms to own public assets (for instance, a

[10] See also Shleifer (1998) on the importance of innovation in ownership choice.

school) and focus their attention on not-for-profit organizations, as they are more easily driven towards public objectives. On the drawbacks of this form of provision, we must remember Dixit (2002b). He shows that these firms might in fact have also additional objectives than the government's ones. The externality they produce by carrying on a public service needs to be taken into account.

The latter contribution (Bennett and Iossa, 2004) is very similar to Hart (2003) in considering building and management as the two main stages of the partnership contract. But PPP is now defined as an ownership structure rather than simple "bundling" of these stages. In addition, they generalize the effects of the builder's investments on the running of the project, in the sense that investments in the first stage have either positive or negative effects on the costs in the second stage. They consider the problem of ownership and closely link it with whether the externality is positive or negative. Given the usual hold up problem causing underinvestment, not internalizing a negative externality (unbundling the stages) may indeed reduce the related inefficiency. With a positive externality, on the contrary, PPP (ownership by a consortium) or public ownership must be preferred.

Several papers consider employment issues[11].We choose to focus in particular on two of them. Francois (2000) introduces "public service motivation" as a possible incentive for workers employed in the public sector. The author develops a more formal approach, based on economic rationality, rather than relying on psychological considerations. In his model, the presence of market incentives may in fact diminish employees' effort. This happens because a profit oriented provider acts as a residual claimant and has an incentive in adjusting inputs to fulfil the contract whenever a worker is underperforming. Even under

[11] See, for instance, Corneo and Rob (2003) or Delfgaauw and Dur (2004). Prendergast (1999) and Dixit (2002a) provide excellent reviews for incentives respectively in private and public organizations.

public service motivation, the worker prefers to shirk as the outcome will still be guaranteed. Relevant elements of the model are the adjustment costs, the degree of substitutability of inputs and the ability to write complete contracts about the outcome. Our model is different, as he deals with employers' decisions about the production quantity rather than with investment choices.

A peculiar approach is developed by Besley and Ghatak (2004). In their model, productivity is increased by a correct match between the mission of an organization and the motivation of its employees. In particular, motivation acts as a substitute of pecuniary incentives and contracting on the mission can provide a firm with more productive and cheaper workers. We decided to develop a more explicit model, without using types as in Besley and Ghatak (2004), but where objective functions of the players may have common elements.

Finally, we must acknowledge the existence of other contributions about privatization and partnerships. These models are less relevant to our purpose as they are either developed in a different framework or focus on different aspects of partnerships (i.e., risk transfer). In our opinion, the most relevant ones are by Grout (1997, 2003) and Bentz, Grout and Halonen (2001). All the contributions are summarized in Table 4.1.

Grout's papers deals with financing and risk issues. In Grout (1997), the author challenges the common opinion that public sector provision necessarily implies lower risk than private provision. The risk is the same even if the government can in fact borrow at a lower rate. This lower rate is not an indication of lower risk, as this is transferred on the general public in form of higher taxes in the future. Then, a standard criterion for implementing PPP projects is that it should be cheaper for the Government to provide a service with a PPP than with public provision. But this criterion raises questions about both the choice of discount factors, in the private and in the public sector, and the reasons for their difference. Starting from a pure finance test, Grout (2003) argues that discount rates used for the public sector should be lower than for the private one. This result is not based on risk arguments, but on the different nature of costs considered for public or partnership provision. It would disappear if the public authority chose to assess partnerships by their construction and maintenance costs rather than by the contracting costs involved.

Table 4.1: Literature review

Topic	**Author**
The incomplete contracts framework	Grossman and Hart (1986) Hart and Moore (1990) Hart (1995)
Public versus private ownership	Hart, Shleifer and Vishny (1997) Besley and Ghatak (2001) Bennett and Iossa (2004)
Partnerships	Grout (1997; 2003) Bentz, Grout and Halonen (2001) Hart (2003)
Workers	Francois (2000) Besley and Ghatak (2004)

Finally, Bentz, Grout and Halonen (2001) adopt a complete contracts approach and focus on the design of incentives from the public to the private sector. The distinction between PPP and public provision is made on the basis of the choice, by the government, respectively to purchase a service or a facility. When investments to make this facility efficient for the service are expensive, than public provision must be preferred. However, when these investments are cheap and delivery costs are low, then PPP is a good choice.

In the next section, we finally present our original models of partnerships.

4.4 Models of PPP

The model we present builds on some previous papers, in particular by Hart (2003), Besley and Ghatak (2004) and Francois (2000). Our contribution is related to the following themes:

- we allow for public provision; to do this, we assume the existence of two stages as in Hart (2003), and that the government does not have the ability to build the facility. There is still a private builder in the first stage but there is room for more possibilities in the second stage. If the provider is public, then we have public provision; if it is a different private firm then we have traditional private provision; finally, if we have a consortium of the builder and the provider, we have a partnership;

- we allow for one additional investment in the second stage; we also allow for the presence of workers in this stage: they can undertake some effort influencing the level of social benefit and are consumers of the good they produce;

- we try to motivate an explicit economic problem and not simply a problem based on "types": motivation refers to how the provision of the service influences the worker's utility.

We divide the problem in two stages: first we allow for workers and public provision; then we introduce some social costs related to the investment in the second stage.

4.4.1 Putting the workers into the picture

Hart (2003) does not consider workers. Besley and Ghatak (2004) do and focus their model on the matching between employers and employees; yet, the relation between workers and employers is based on types and not on explicit utility functions, as in our model. Finally, Francois (2000) is closer to our contribution in making his "public service motivation" an economical problem. Nevertheless, his model deals with employers' decisions about the production quantity rather than with investments choices.

Our aim is to introduce two corrections in Hart's model. The first one is about the presence of workers, as we believe their contribution is not irrelevant for the success of the good or service under provision. The second one is about the possibility for the provider to undertake some investments. In order to understand the relevance of these two new elements, an example might be useful. Let's consider a radiologist in a hospital. His ability to diagnose an illness may depend on the accuracy of some exam: without, for instance, an X-ray machine, he is not able to understand the problem of his patient. The choice of buying or not this kind of machine is an investment bearing on the provider's side. Finally, costlier machines provide more accurate results, which are easier to interpret for the radiologist. In other words, there is some complementarity between the investment of the employer and the effort of the worker. Both of them can influence the quality of the service which is provided (in this case, health services).

We also want to differentiate our model from Besley and Ghatak (2004) by making incentives for workers more explicit, without relying on matching of exogenous types or psychological motivations. The worker can find some satisfaction in his job, which is higher the greater his contribution to the final result. He may also enjoy the service himself as a consumer. For instance, a teacher, whose children go to the school where he works, might have an incentive to improve the level of teaching.

We now present a first version of the model, with a single worker. First we work out the efficient levels of investments, as a benchmark case. Then, we recall Hart's results (Hart, 2003) which we are going to use. Finally, we discuss our original contribution to his model.

The setting

There are four subjects: a builder B, a private provider P, the government G, and a worker L. The government G is not a social welfare maximizer agent *strictu sensu*[12], but only cares about the social effect of the service. This is measured by a social benefit function SB rather than by the sum of the agents' utility and profit functions (worker, government itself and firms). G could also be a public agency. In reality, this is the most natural interpretation. We therefore need to assume that the objective functions of the government and of its agency are the same. The production of the service requires two stages: a building one and an operating one. In the former stage a facility or a capital asset is built; this is used in the latter stage to actually provide the good or service. For instance, the facility can be a school, a hospital or a prison. The building stage is technologically impossible for G, so it must contract it out to B. Before this contract is signed, G must also

[12] As it will become clear below (see Lemma 5 and Lemma 8), as long as we do not consider any social cost caused by the provider's investment, G eventually acts as a social welfare maximizer even if it is not defined as such.

decide which kind of provider will operate the facility. G has three choices: it can operate the facility itself (public provision), it can contract out the provision to an independent private provider P (traditional private provision), or it can contract it out to the builder itself. The builder can then subcontract the operational stage to P or form with him a consortium. We refer to the latter as a PPP and we assume, for simplicity and without loss of generality, that a partnership is precisely formed by a consortium of B and P. The worker L is employed in the second stage: therefore he is a public employee under public provision and a private employee under private provision or PPP.

All the subjects can undertake some non contractible effort or investments: exactly as in Hart (2003), the builder can decide whether to invest in i, a productive investment increasing SB and decreasing operational costs OC for the provider (e.g., a nicer building which is also easier to operate), and in e, an unproductive investment, decreasing both SB and OC (a facility may be easier to operate but less safe to its users). We add two more investments. First, the provider can invest in a, a productive investment with a positive effect on SB (for instance, a better or more powerful machine in a hospital or teaching instruments). Finally, L can undertake some effort on his job. This effort d increases the quality, or level, of the service which is provided. Recalling the example about the radiologist, it is clear that the returns from these two investments, a and d, are mutually dependent: the result of a medical test needs an interpretation to become a diagnosis; likewise, the radiologist's opinion needs to be funded on solid bases.

The utility functions of the agents under different provision forms are the following:

Private provision	Public provision	Partnership
$U_G^p = SB - \lambda_B - \lambda_P$	$U_G^g = SB - \lambda_B - OC$	$U_G^{ppp} = SB - \lambda_{PPP}$
$\Pi_B = \lambda_B - i - e$	$\Pi_B = \lambda_B - i - e$	$\Pi_{PPP} = \lambda_{PPP} - i - e - OC$
$\Pi_P = \lambda_P - OC$		

and

$$U_L = w + \delta f(a,d) - d$$

with

$$\begin{aligned} SB &= B_0 + b(i) - \beta(e) + f(a,d) \\ OC &= C_0 - c(i) - \gamma(e) + w + a \end{aligned}$$

The utility of G (U_G) is given by SB minus the price (or prices) λ, paid to the private firms. Under traditional private provision, G pays λ_B to the builder and λ_P to the provider; under partnership, G pays λ_{PPP} to the consortium. Finally, under public provision, G pays λ_B to the builder and directly suffers the operational costs OC. We assume a fully competitive market for the firms. Therefore, they are paid competitive prices by G and their profits are simply driven to the market ones, which we assume equal to 0. More specifically, the profit of the builder, Π_B, is given by the competitive price paid by G for its service, λ_B, minus the level of investments he decides to undertake in equilibrium. The provider P is paid a competitive price λ_P by G and incur in some operational costs, OC. If a partnership is formed, then it is paid a price λ_{ppp} by G and incurs in all the costs of the service, that is i, e and OC. Finally, the worker's utility (U_L) depends on his wage w, the level of his effort d, and on the quality, or level, of the service which is provided. This is particularly true when the good is a public or a collective one, as it is in our model. The parameter δ, with $\delta \in (0,1)$, may measure the sensitivity of the worker to the service. For instance, local workers might be more affected if they use the service they provide. We can interpret δ also as a characteristic of the good itself, that is, the degree of its public dimension. Finally, δ can reflect the worker's commitment to the public service. The worker's effort d directly increases SB through $f(a,d)$ and indirectly increases the worker's own utility. The marginal contribution of d is increasing in a, that is, $\frac{\partial^2 f(a,d)}{\partial a \partial d} > 0$, and, in particular, without any investment in a, d has no effect: $f(0,d) = 0$ $\forall d$. Recalling the previous example, a radiologist might find it impossible to interpret some analysis if the machine he uses is not good enough. Finally, the worker has a reservation utility from not working equal to $\bar{U}$.

The functions $\beta(e)$, $c(i)$, $\gamma(e)$, $b(i)$ and $f(a,d)$ embody the effects of the various investments on SB, OC and U_L. The social benefit function, that we defined as the social effect of the service provided, is positively affected both by the productive investment i, undertaken by the builder, and jointly by a and d. The unproductive investment e has a negative effect on the social benefit. As regards the operational costs OC, they decrease with both the builder's investments, i and e. They also include the provider's investment, a, and the wage w of the worker, as he is employed by the provider.

The function $f(a,d)$ appears twice in the payoffs functions. The first time as a positive contribution to SB. Both a and d positively affect the quality of the service. The second time, $f(a,d)$ appears as a positive effect on the worker's utility. We can interpret this effect in different ways. The most straightforward one is the presence of satisfaction from the final result of the job. A better result, which depends both on a and d, can make the worker happier about himself. A second interpretation is that the worker can be a consumer himself, therefore enjoying a share of the public or collective good that he helps to provide.

For technical reasons, we assume $f(a,d)$ to be a Cobb-Douglas production function with decreasing return of scale[13]; that is:

$$f(a,d) = a^{\alpha} d^{\beta}$$

with:

$$\alpha, \beta \in (0,1)$$

$$\alpha + \beta < 1$$

If not strictly necessary to our computations, through the chapter we will refer to $f(a,d)$ in its generic expression. Finally, further technical assumptions about the functions above apply. All the functions are non negative and increasing in their arguments. The function $\beta(e)$, which decreases the social benefit, is strictly convex, whereas functions increasing profits and utilities $(c(i); \gamma(e); b(i); f(a,d))$ are strictly concave in their arguments. We further assume that $f(a,d)$ satisfies Inada conditions.

[13] In this way, we ensure the existence of an internal equilibrium and we are able to provide explicit comparisons among solutions.

The benchmark case

Efficiency requires the maximization of the social welfare function (SWF), which we define as the sum of all the agents' payoffs. Taking into account all the cross payments, this is equal to:

$$SWF = B_0 + b(i) - \beta(e) + (1+\delta)f(a,d) - C_0 + c(i) + \gamma(e) - i - e - a - d, \tag{4.1}$$

hence the social welfare maximizer's problem:

$$\max_{i,e,a,d} SWF$$

with non-negativity constraints:

$$i, e, a, d \geq 0$$

The first order conditions are:

$$\begin{cases} [b_i(i^*) + c_i(i^*) - 1]\, i^* = 0 \\ [-\beta_e(e^*) + \gamma_e(e^*) - 1]\, e^* = 0 \\ [(1+\delta)f_a(a^*, d^*) - 1]\, a^* = 0 \\ [(1+\delta)f_d(a^*, d^*) - 1]\, d^* = 0 \end{cases} \tag{4.2}$$

where $h_j(j) = \dfrac{\partial h(j)}{\partial j}$ and j^* is the efficient level of investment j.

We are keeping Hart's assumptions about the solutions for i and e. These assumptions refer to the existence of an interior solution for i and a corner one for e (that is, $\gamma_e(0) -$

$\beta_e(0) - 1 \leq 0$). In addition, we assume that both a and d have a internal solutions as well:

$$\begin{cases} b_i(i^*) + c_i(i^*) - 1 = 0 \\ e^* = 0 \\ (1+\delta) f_a(a^*, d^*) - 1 = 0 \\ (1+\delta) f_d(a^*, d^*) - 1 = 0 \end{cases}$$

Before analyzing the worker's contribution, we recall Hart's results (Hart, 2003) and highlight the relationships between our models.

Hart (2003)

In this model, the government can choose only between traditional private provision and PPPs. In the former case, the builder has no incentive in investing either in i or e, as he does not consider the effects of these investments on the provider's operational costs OC and on the social benefit SB. With a partnership between the builder and the provider, the external effects of e and i on OC are taken into account, but not the effect on SB. The result is that a partnership will provide a more efficient level of investment i but a less efficient level of investment e. As far as investments in i and e are concerned, no difference emerges between our two models. The worker worries only about the level of a and d, which are not present in Hart (2003). In this way, comparisons between the models are easier. In addition, the level of investments i and e reached under traditional private provision are the same that we reach under public provision.

We can formalize this discussion, in order to better understand his results. At this point, we focus only on investments in the first stage: i and e. We label the investments under traditional private provision, under partnerships and under public provision respectively as i_p, i_{ppp}, i_g and e_p, e_{ppp}, e_g.

Investments under private provision Under private provision, the government signs two short-term contracts with two different private firms: B and P. The problem of the builder is therefore the following:

$$\max_{i,e} \Pi_B = \lambda_B - i - e$$

In the second period, the problem of the provider is:

$$\max_{a} \Pi_P = \lambda_P - OC$$

With the resulting investments:

$$i_p = e_p = 0$$

None of the firms has an incentive in investing: the builder considers i and e simply as a loss, as they do not influence its costs or productivity.

Investments under partnership When a partnership between B and P is formed, the consortium takes into account the effects of i and e on the provider's costs. The governments signs one long term contract with this consortium, whose problems is:

$$\max_{i,e,a} \Pi_{PPP} = \lambda_{PPP} - OC - i - e$$

With the resulting investments:

$$\begin{cases} c_i(i_{ppp}) = 1, \\ \gamma_e(e_{ppp}) = 1, \end{cases}$$

In this case the consortium takes into account the effect of i and e on the provider's costs. As regards i, this is still insufficient, as no weight is given to the social benefit. As for e, the lack of consideration for its negative effect on social benefits leads to an overinvestment.

Investments under public provision Finally, with public provision the government signs only one short term contract with the builder and then provides the service or good in-house. The problem of the builder is the same as with private provision:

$$\max_{i,e} \Pi_B = \lambda_B - i - e$$

In the second period, the public provider solves the following problem:

$$\max_a U_G = SB - \lambda_B - OC$$

With the resulting investments:

$$i_g = e_g = 0$$

The builder does not make any difference about the identity of the provider, as long as it is not part of a consortium. Therefore no investment in i and e will B's choice, as the investment in i is not contractible.

Proposition 12 is based on these results, which will be restated in Proposition 13 in the light of our original contribution.

Proposition 12 (Hart, 2003 revisited) *We can order the levels of investments reached under different provision mechanisms. We have that:*

$$i_p = i_g < i_{ppp} < i^*$$

$$e^* = e_p = e_g < e_{ppp}$$

Traditional private provision and public provision (as defined above) are better when the quality of the building can be easily detailed but the quality of the service cannot. In other words, when it is relatively easier to write the contract about the quality of the building, private provision or public provision minimize the inefficiency. The underinvestment in "i" is not a serious issue and investment in "e" is even efficient. Mutatis mutandis, partnerships should be preferred when it is easier to measure or specify the quality of the service in the contract.

Proof. See Hart (2003) and the discussion above. ■

We now present a model where the additional investments (by the provider and the worker) can lead to different choices. We first introduce a setting with general functions, and then we provide a more specific example.

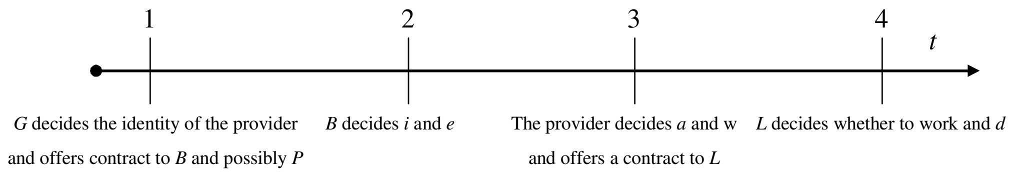
Private versus public employment

We can now compare public and private employment in a second best setting. To do this, it is important to clarify a time-line of decisions in this economy (Graph 4.1).

At $t = 1$, G decides what kind of provision it wants (public, private or a partnership).

Consequently, G also decides the identity of $L's$ employer and signs a particular contract: with public or private provision this would be a short term contract with the builder B about the quality of the building; with a partnership, this would be a long term contract with the consortium of B and P about the quality of the service. At $t = 2$, the builder decides the level of investments i and e. At $t = 3$, the provider decides the level of investment a and the level of the wage w; in case of private provision, G previously signs a short term contract with P about the quality of the service Finally, at $t = 4$ the worker observes a and w. Then, he decides whether to accept the job or not and, in the positive case, what level of effort d to undertake.

Graph 4.1: Time line



All these investments decisions are *ex post* observable, but are not verifiable. So, it is not possible to write complete contracts and set up a system of payments and transfers such that the efficient investment levels are realized.

Having already shown the results for i and e, we can now focus only on the provider's problem with respect to w and a. The model is solved as a typical principal-agent one and by backwards induction, starting from $t = 4$. It is important to stress that the worker's problem is independent of the identity of his employer (whereas its choice is not, as we will show later). We also notice that no difference arises between private provision and a partnership. Therefore we analyze both cases together.

At $t = 4$, L solves the following problem:

$$\max_{d} w + \delta f(a,d) - d$$
$$\text{s.t. } d \geq 0$$

Assuming $f(a,d) = (a^{\alpha} d^{\beta})$, we obtain the following first order condition:

$$\delta \beta a^{\alpha} d^{\beta - 1} = 1 \tag{4.3}$$

We can rearrange the terms so that we have:

$$d = g(a) = (\delta a^{\alpha} \beta)^{\frac{1}{1-\beta}} \tag{4.4}$$

where $g(a)$ denotes the reaction function for the worker's effort. The worker's choice is therefore between working, and reacting to a according to (4.4), and not working, if the incentive provided by the employer is not high enough:

$$\max \left\{ w + \delta f(a, g(a)) - g(a), \bar{U} \right\}$$

In other words, the worker will accept a contract only if :

$$w + \delta f(a, g(a)) - g(a) \geq \bar{U}$$

The relationship between a and d is used by the principal at $t = 3$. We now analyze the employer's problem by discriminating between private and public provision. From the first order conditions of his problem, we already know that, for fixed values of a and δ, d is inefficiently low, as the worker takes into account only his private benefit and not the entire social one. With an abuse of notation, we call the worker's socially optimal reaction function $g^*(a)$. So we can state that:

$$d = g(a) < d^* = g^*(a) \tag{4.5}$$

and:

$$f(a, d) < f(a, d^*) \tag{4.6}$$

given our assumption on $f(\bullet)$ that $f_{ad}(a, d) > 0$ and $f_{aa}(a, d) < 0$.

Private provision. At $t = 3$, the private provider P (the consortium would face a similar situation: see proof of Lemma 6 below) must solve the following problem: the firm wants to minimize its costs but has to consider the worker's constraints. The individual rationality (or participation) constraint IR states that the worker will accept the job only if it provides him at least his reservation utility $\bar{U}$. The incentive compatibility constraint (IC) states that the action the worker will take on the job must be optimal from his point of view, and directly follows from the first order conditions of his maximization problem:

$$\begin{aligned} \min_{a,w} OC &= C_0 - c(i) - \gamma(e) + w + a \\ \text{s.t.:} \quad w + \delta(a^\alpha d^\beta) - d &\geq \bar{U} \quad IR \\ d = g(a) &= (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} \quad IC \end{aligned}$$

where i and e were chosen at $t = 2$, and therefore $c(i)$ and $\gamma(e)$ are treated as constant terms.

The worker's participation constraint is binding in equilibrium, as otherwise the principal would have an incentive to decrease w. The provider has indeed two possibilities to attract the worker, and can choose the cheapest one: it can either invest in some minimum level of a or it can raise the wage, such that the reservation utility is equalized. We can first substitute IC into IR, and then, knowing that the latter is binding, we can directly substitute for w in the objective function. The principal's problem (in a simplified version) is then only one dimensional:

$$\min_a \left\{ \bar{U} - \delta a^{\alpha} \left[(\delta a^{\alpha} \beta)^{\frac{1}{1-\beta}} \right]^{\beta} + a + (\delta a^{\alpha} \beta)^{\frac{1}{1-\beta}} \right\} \tag{4.7}$$

The first order condition of the problem is:

$$1 - \beta + a^{-1} \alpha \left(\delta a^{\alpha} \beta \right)^{\frac{1}{1-\beta}} - \alpha \delta a^{\alpha-1} \left[(\delta a^{\alpha} \beta)^{\frac{1}{1-\beta}} \right]^{\beta} = 0 \tag{4.8}$$

The principal determines the value a_p such that:

$$a_p \in \arg\min OC$$

In this case, we have that:

$$a_p = \left[\delta \alpha^{1-\beta} \beta^{\beta} \right]^{\frac{1}{1-\alpha-\beta}} \tag{4.9}$$

Finally, the wage in the private sector ($w_p = w_{ppp}$) is easily determined from the worker's IC :

$$w_p = \bar{U} - \delta f(a_p, d_p) + d_p$$

Public provision. With respect to the worker, the government must solve the following (simplified)[14] problem:

$$\max_{a,w} U_G = (a^\alpha d^\beta) - a - w$$

$$\text{s.t.:} \quad w + \delta(a^\alpha d^\beta) - d \geq \bar{U} \quad IR$$

$$d = g(a) = (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} \quad IC$$

Applying the same logic as above, we reduce the government's problem to the following:

$$\max_a \left\{(1+\delta)a^\alpha \left[(\delta a^\alpha \beta)^{\frac{1}{1-\beta}}\right]^\beta - a - (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} - \bar{U}\right\}$$

whose first order condition is:

$$1 - \beta + \alpha a^{-1} (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} - \alpha(1+\delta)a^{\alpha-1} \left[(\delta a^\alpha \beta)^{\frac{1}{1-\beta}}\right]^\beta = 0 \tag{4.10}$$

Finally, the government determines the value a_g such that:

$$a_g \in \arg\max U_G$$

In this case, we have that:

$$a_g = \left[\delta^\beta \alpha^{1-\beta} \beta^\beta\right]^{\frac{1}{1-\alpha-\beta}} \left[\frac{1-\beta}{(1+\delta-\delta\beta)}\right]^{\frac{\beta-1}{1-\alpha-\beta}} \tag{4.11}$$

[14] We are ignoring the terms $c(i)$ and $\gamma(e)$ as the government has no control over them.

Finally, the wage in the public sector (w_g) is easily determined from the worker's IC :

$$w_g = \bar{U} - \delta f(a_g, d_g) + d_g$$

Comments, comparison and an example. These results allow some initial comment. Using Hart's approach (Hart, 2003), we can state that if the characteristics of the facility are easier to specify, the government should provide the service. If, on the contrary, the quality of the service is easier to measure, then the choice is between public provision and partnerships. Public provision is preferred when worker's effort is very relevant for the success of the service.

We now formalize our findings with Proposition 13, which updates Proposition 12, and then comment on it.

Proposition 13 *We can compare the levels of investments provided under different provision choices. We have that:*

$$\begin{cases} i_p = i_g < i_{ppp} < i^* \\ e^* = e_p = e_g < e_{ppp} \\ a_{ppp} = a_p < a_g < a^* \\ d_{ppp} = d_p < d_g < d^* \end{cases} \tag{4.12}$$

When we allow for public provision, for workers' efforts and investments in the second stage, private provision is always dominated by public provision. Investments in i and e are the same, but investments in a and d are closer to the efficient level.

Proof. The first two rows in (4.12) are part of Proposition 12 and have been proven above.

As regards, a and d, it follows from direct comparison of $a_{ppp} = a_p$ in (4.9) and a_g in (4.11) that $a_{ppp} = a_p < a_g$. The efficient value of a, a^*, can be worked out by substituting $f(a, d) = a^{\alpha} d^{\beta}$ in the social planner's problem. We obtain that:

$$a^* = \left[(1+\delta)\alpha^{1-\beta}\beta^{\beta}\right]^{\frac{1}{1-\alpha-\beta}}$$

By comparing a^* and a_g, we obtain the following condition:

$$\begin{gathered} a_g < a^* \quad \text{iff} \\ \delta^{\beta}\left(\frac{1+\delta(1-\beta)}{1-\beta}\right)^{1-\beta} < 1+\delta \end{gathered} \tag{4.13}$$

For the range of values of the parameters under consideration[15], this condition is always satisfied[16] (equality holds for $\beta = 0$, but we do not consider this case). So we can state that:

$$a_{ppp} = a_p < a_g < a^* \tag{4.14}$$

Given (4.14), it must follow that:

$$d_{ppp} = d_p < d_g < d^* \tag{4.15}$$

The reason for the result in (4.15) is quite intuitive. In second best (public and private provision), the worker's reaction function $g(a)$ is the same; therefore, if $a_p < a_g$, then it is straightforward that $d_p < d_g$. Moreover, from the social planner's problem, we can obtain

[15] That is, $\beta \in (0, 1)$ and $\delta \in (0, 1)$

[16] The condition in (4.13) is formally proven in Appendix.

the efficient reaction function $g^*(a)$:

$$g^*(a) = [(1+\delta)a^{\alpha}\beta]^{\frac{1}{1-\beta}}$$

This is such that:

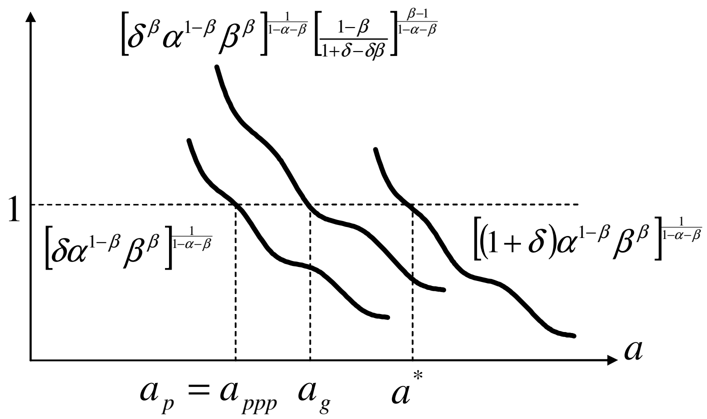
$$g(a) < g^*(a) \quad \forall a > 0,$$

and so it is *a fortiori* true that, if $a_g < a^*$, then $d_g < d^*$.

As regards the result in (4.14), the intuition is that the public provider correctly takes into account the full benefit from a and d. Nevertheless, this is not enough to produce an efficient level of a, as $g(a) < g^*(a)$. In other words, for any unit of investment a which is undertaken by G, the return (social benefit) is smaller than in the efficiency case, as the worker's contribution ($g(a)$) is inefficiently low.

In Graph 4.2, we show the ranking of the investments in a under different provision mechanisms.

Graph 4.2: The choice of a



The government takes correctly into account both the social benefit and the worker's private benefit from the good. Nevertheless, we know from (4.6) that the benefits are definitely smaller than in the first best: both a_g and d_g are set at an inefficient level.

The best choice is the one maximizing the SWF, as defined in (4.1). In Table 4.2 we compare the values taken by the SWF under different provision schemes. The first best is never reached; from comparisons in (4.12) and in Table 4.2, and recalling Proposition 12, Proposition 13 directly follows.

Table 4.2: The optimal choice

Private provision:	SWF^p :	$B_0 + (1+\delta)f(a_p, d_p) - C_0 - a_p - d_p$
Public provision:	SWF^g :	$B_0 + (1+\delta)f(a_g, d_g) - C_0 - a_g - d_g$
Partnership	SWF^{ppp} :	$B_0 + b(i_{ppp}) - \beta(e_{ppp}) + (1+\delta)f(a_p, d_p) - C_0 + c(i_{ppp})$
		$+\gamma(e_{ppp}) - i_{ppp} - e_{ppp} - a_p - d_p$

■

We want to focus our comments in particular on a and d. As long as no social cost is associated to the investment in the second stage (a), public provision is always the best provision choice. The government is interested in maximizing the social benefit, net of its provision costs, and, even if it does not take into account all the benefits from a, its choice is the closest to the first-best. But this also means that public provision is better *as long as* a provider can influence the quality of the service. The importance of the investment is summarized by the shape of the function $f(\bullet)$.

If we think about health services, for instance, investments in the provision stage could be expensive and technological machinery. Even if a private provider could afford such an investment, it is not straightforward that he would undertake it. In education, these kinds of expensive investments seem less frequent. Most of the issues usually regard the quality of the building, something that the government cannot really influence. Our model would therefore suggest that the health service should be publicly provided whereas education could be left to the private sector.

Nevertheless, if we consider also the role of i and e, partnership may be better than public provision. This is particularly true when the reduction in operational costs associated to these investments is higher than the corresponding reduction in the social benefit. Hart (2003) reckons that health service should be provided under a PPP. Our model suggests that the reduction in operational costs should offset the reduction in social benefit following not only higher investments in i and e, but also lower investments in a and d. Recalling how we stress the importance of complementarity between a and d, our model may actually support the opinion that some of the staff should be privately occupied and part of the staff publicly employed. In particular, the public sector should retain its control over workers whose contribution is very high to the level of the service and leave the others to the private sector.

The government is facing two kinds of choices: an *investment* choice (i.e.: the level of a) and a *provision* choice (i.e.: the identity of the provider). The former choice induce distorted level of investments, as we showed above. What about the latter? The maximization of the SWF gives us a normative criterion to decide which provider is better. Nevertheless, in reality governments are not social welfare maximizers but have their own utility functions. In our model, we allow the government to have social concerns by considering SB as part of its maximization problem. Nevertheless, U_G does not consider all the utilities (or profit functions) of every agent in the economy. This may lead to a clash between what is best for society (i.e., a provision choice according to the SWF) and what is actually implemented (i.e.: a provision choice according to U_G, the utility function of the decision maker).

In other words, as the government is maximizing U_G and not SWF, it would choose according to the resulting investments maximizing its utility function rather than the social welfare function. Lemma 5 solves this doubt.

Lemma 5 *There is no distortion in the second best "provision" choice by the government.*

Proof. We can compare the government's actual choice, based on U_G, evaluated for different provision choices, and the optimal one, based on SWF, evaluated in the same values. We summarize this problem in Table 4.3.

Table 4.3: The government's choice

		U_G
Private provision:	U_G^p :	$SB - \lambda_B - \lambda_P = B_0 + (1+\delta)f(a_p, d_p) - C_0 - a_p - d_p - \bar{U}$
Public provision:	U_G^g :	$SB - \lambda_B - OC = B_0 + (1+\delta)f(a_g, d_g) - C_0 - a_g - d_g - \bar{U}$
Partnership	U_G^{ppp} :	$SB - \lambda = B_0 + b(i_{ppp}) - \beta(e_{ppp}) + (1+\delta)f(a_p, d_p) - C_0$
		$+c(i_{ppp}) + \gamma(e_{ppp}) - i_{ppp} - e_{ppp} - a_p - d_p - \bar{U}$

As $\bar{U}$ is simply a constant, the government faces the same problem as a hypothetical social welfare maximizer. Therefore its provision choice is not distorted. When a partnership is better than public provision according to a SWF criterion, then it is better also according to G's maximization problem. And so it is for the other way round. ■

The government is forced to consider the other agents' profit and utility functions, as he is the ultimate payer for the service. Under public provision, its choice of a is not optimal just because the worker's reaction function is distorted.

As regards the role of workers, the fact that they are consumers makes them indirectly sympathetic with the government's preferences. Public provision is therefore better as workers' and government's efforts are complementary to the success of the service. Again, a similar caveat regarding the shape of $d(\bullet)$ is worth of mention. Public employment is better *as long as* the workers' contribution to the quality of the service is relevant. As suggested above, this result might also justify the case for mixed employment schemes. For instance, in the case of public transport, our model may suggest that buses designers should be public employees whereas it doesn't really matter who hires, for instance, the administrative staff.

Additional comments follow the example below.

Example 1 *In this example, we only focus on the worker's problem about "d" and the principal's choice about "a". In other words, we completely ignore issues about the remaining investments "i" and "e". We assume that* $\alpha = \beta = \frac{1}{3}$*:*

$$f(a, d) = (ad)^{\frac{1}{3}}$$

that is, the contribution of the two investments is symmetric. The FOCs in the first best case are:

$$\begin{cases} \frac{\partial SWF}{\partial a} : a = (\frac{1+\delta}{3})^{\frac{3}{2}} \sqrt{d} \\ \frac{\partial SWF}{\partial d} : d = (\frac{1+\delta}{3})^{\frac{3}{2}} \sqrt{a} \end{cases}$$

with interior solutions:

$$a^* = d^* = (\frac{1+\delta}{3})^3$$

The worker's reaction function $g(a)$ is:

$$d = (\frac{\delta}{3})^{\frac{3}{2}}\sqrt{a} < (\frac{1+\delta}{3})^{\frac{3}{2}}\sqrt{a} \text{ for any } a > 0$$

When we solve for the private firm's and the government's optimal a, we obtain respectively:

$$\begin{cases} a_p = (\frac{\delta}{3})^3 < (\frac{1+\delta}{3})^3 \\ a_g = \frac{\delta}{3}(\frac{3+2\delta}{6})^2 < (\frac{1+\delta}{3})^3 \end{cases}$$

with:

$$a^* > a_g > a_p$$

The corresponding worker's effort are respectively:

$$\begin{cases} d_p = (\frac{\delta}{3})^3 < (\frac{1+\delta}{3})^3 \\ d_g = (\frac{\delta}{3})^2(\frac{3+2\delta}{6}) < (\frac{1+\delta}{3})^3 \end{cases}$$

with:

$$d^* > d_g > d_p$$

Finally, we can determine wages; recalling that in equilibrium:

$$w = \bar{U} - \delta f(a, d) + d$$

we have:

$$\begin{cases} w_p = \bar{U} - 2(\frac{\delta}{3})^3 \\ w_g = \bar{U} - 2(\frac{\delta}{3})^2(\frac{3+2\delta}{6}) \end{cases} \tag{4.16}$$

so that:

$$w_g \leq w_p$$

Example 1 gives us the opportunity to draw some additional considerations about wages in the private and the public sector, but has no direct relevance for the PPP debate.

From (4.16), we know that if $\delta = 0$, there is no reason why we should observe wage differentials in the private and in the public sector. We introduced δ as a characteristic of the worker, who is also the consumer of the good he produces. We also offered a different interpretation of δ, that is, the degree of public dimension of the good itself. In public and private firms producing the same kind of pure private good, we expect workers to demand the same wage. If they do not, it is probably because of other dimensions of the problem (e.g.: job security, retirement plans). What our model explains is why people accept similar jobs in the public sector for lower monetary incentives (see also, for example, Dixit, 2002a and 2002b). They do so as long as they believe the can cooperate with their employer to satisfy their preferences.

The general result is stated and proven in Lemma 6.

Lemma 6 *The wage of the worker in the private sector is never smaller than the wage in the public sector:*

$$w_g \leq w_p = w_{ppp} \tag{4.17}$$

Proof. First of all, we show that $w_p = w_{ppp}$. As explained above, at $t = 3$ a private provider P solves the following problem:

$$\min_a \left\{ \bar{U} - \delta f(a, g(a)) + a + g(a) + C_0 \right\}$$

whereas a consortium has a slightly different cost function:

$$\min_a \left\{ \bar{U} - \delta f(a, g(a)) + a + g(a) + C_0 - c(i_{ppp}) - \gamma(e_{ppp}) + i_{ppp} + e_{ppp} \right\}$$

The difference emerges as, with traditional private provision, P cannot control i and e. But, as at $t = 3$ both the private provider P and the consortium only choose a, their solution is the same, and so it is the wage they pay to L.

Then, we know that:

$$w_g = \bar{U} - \delta f(a_g, d_g) + d_g$$

$$w_p = \bar{U} - \delta f(a_p, d_p) + d_p$$

and that:

$$a_p < a_g; d_p < d_g$$

so

$$f(a_g, d_g) > f(a_p, d_p)$$

The function $f(a, d)$ is increasing in a by assumption. This means that:

$$f(a_g, d_p) > f(a_p, d_p)$$

The utility $\delta f(a, d) - d$ that a public worker obtains from observing a_g and producing d_g must be bigger than the his utility from observing a_g and producing d_p. Otherwise, he would be better off by decreasing his effort level to d_p. Therefore:

$$\delta f(a_g, d_g) - d_g > \delta f(a_p, d_p) - d_p$$

and (4.17) follows. ■

Under a different point of view, the model can explain why people are happy to work for free in charities like Oxfam but they require a wage from, say, Blackwell's. Though the *economic* activity is the same (selling books), people working in Oxfam probably have some preferences for the *social* activity of the charity.

This conclusion is extremely similar to the one in Besley and Ghatak (2004), except for the fact that we do not model our workers using "types" but explicit utility functions. G and L have, at least partially, the same objective, which is the quality of the service. This argument can be expressed in Besley and Ghatak's terms as: "G and L are of the same type".

The choice under budget constraint

A final remark about the (second) best provision choice is worthy of mention. So far we have implicitly assumed that G could spend any amount of money, that is, its choice was unconstrained. It can be interesting to compare the three provision choices purely in terms of their cost to the government, that is, without considering the social benefit they imply.

Let's define TC as the total cost function for the government. Under different provision mechanisms, we have three possible cases:

$$TC^p = \lambda_B + \lambda_P$$

$$TC^{ppp} = \lambda_{PPP}$$

$$TC^{g} = \lambda_B + OC$$

where:

$$\lambda_B = i_p + e_p$$

$$\lambda_P = C_0 - c(i_p) - \gamma(e_p) + a_p + w$$

$$\lambda_{PPP} = C_0 - c(i_{ppp}) - \gamma(e_{ppp}) + a_{ppp} + w$$

and:

$$w = \bar{U} - \delta f(a, d) + d$$

Under private and public provision, $\lambda_B = 0$, as $i_p = e_p = 0$. Following the analysis in the previous sections, and recalling that $a_p = a_{ppp}$ and $d_p = d_{ppp}$, we can conclude that:

$$TC^{p} = C_0 + a_p + \bar{U} - \delta f(a_p, d_p) + d_p \tag{4.18}$$

$$TC^{ppp} = C_0 + i_{ppp} + e_{ppp} - c(i_{ppp}) - \gamma(e_{ppp}) + a_p + \bar{U} - \delta f(a_p, d_p) + d_p \tag{4.19}$$

$$TC^{g} = C_0 + a_g + \bar{U} - \delta f(a_g, d_g) + d_g \tag{4.20}$$

Lemma 7 directly follows.

Lemma 7 *Partnerships are the cheapest provision mechanism:* $TC^{ppp} < TC^{p} < TC^{g}$

Proof. It is straightforward to see that $TC^{ppp} < TC^{p}$ by direct comparison of (4.18) and (4.19). We have that:

$$TC^{ppp} = TC^{p} + i_{ppp} + e_{ppp} - c(i_{ppp}) - \gamma(e_{ppp})$$

The quantity $c(i_{ppp}) + \gamma(e_{ppp}) - i_{ppp} - e_{ppp}$ must be positive for internal solutions of i and e, as it is maximized by the partnership. Therefore $TC^{ppp} < TC^{p}$.

As regards $TC^{p} < TC^{g}$, the comparison reduces to the following:

$$a_p - \delta f(a_p, d_p) + d_p \lesseqgtr a_g - \delta f(a_g, d_g) + d_g$$

The left hand side of the inequality corresponds to the private provider's minimization problem, as expressed in (4.7). Therefore, it is at its minimum exactly when $a = a_p$. So the right hand side must be bigger. ■

When G hires a partnership, it incurs the lowest possible price λ_{PPP} for the service. A partnership obtains operational costs savings which neither a public nor a private provider can realize. The private sector pays a higher wage, but this wage is not the only labour cost. Both the public and the private sector must give their worker a level of utility equal to $\bar{U}$. What the government saves in w must be paid through a higher investment in a. Our model suggests that private provision (traditional or partnership) is a good way of realizing savings for the government and confirms the common opinion that private provision is cheaper. Whether it is also preferable from a social welfare point of view is not always clear, as stated in Proposition 13.

Partnerships are chosen because they provide "value for money": this criterion is always realized when the government does not have a budget constraint. If G is bounded to spend no more than some level $TC < TC^g$, then privatization might be an inefficient solution, though the only available one.

4.4.2 A wider choice

In this subsection, we want to show that the scope for public provision is not limited by the presence of workers. Efficiency or inefficiency of public provision can have also a different source. In order to do this, we introduce some social costs associated to the investment in the second stage (a), with the aim of compensating the previous bias against traditional private provision. As anticipated, we completely ignore the role of workers. The rest of the setting is the same as in the previous section. The main difference concerns the government's utility function, which is now equal to:

$$U_G = \theta SB + (1 - \theta)PB$$

where:

$$SB = B_0 + b(i) - \beta(e) + f(a)$$

is the social benefit given by the investments in the production (building and running) process; and

$$PB = n(a)$$

is the private benefit for the public provider. PB can have the following interpretation: $n(a)$ is the probability of being re-elected due to the votes-catching investment a; a has a positive effect on the collective benefit, but might be used also to obtain electoral consensus (e.g., through excessive employment).

The parameter $(1-\theta)$, with $\theta \in (0,1)$, can be interpreted as the degree of corruptibility of the government. In the previous subsection, we simply assumed $\theta = 1$.

We further assume that the private benefit PB is a complete waste for the society; that is, an investment of a in the second stage creates an increase in the social benefit equal to $f(a)$ but also a social cost (excessive staff, propaganda) equal to $n(a)$. Therefore:

$$SC = n(a)$$

The private firms' objective functions are unchanged:

$$\begin{aligned}
\Pi_B &= \lambda_B - i - e \\
\Pi_P &= \lambda_P - OC \\
\Pi_{PPP} &= \lambda_{PPP} - i - e - OC
\end{aligned}$$

with

$$OC = C_0 - c(i) - \gamma(e) + a$$

As before, the profit of the builder, Π_B, is given by the competitive price paid by G for its service, λ_B, minus the level of investments he decides to undertake in equilibrium. The provider P is paid a competitive price λ_P by G and incur in some operational costs, OC. If a partnership is formed, then it is paid a price λ_{PPP} by G and incurs in all the costs of the service, that is i, e and OC.

All the previous assumptions about the functions above still apply. In particular, we recall that $\beta(e)$ is strictly convex whereas $c(i), \gamma(e), b(i), f(a)$ and also $n(a)$ are strictly concave.

Efficiency requires the maximization of the following social welfare function, which is the algebraic sum of objective functions of the subjects in this economy and of the social costs function. Once taken into account all the cross payments, this is equal to:

$$\begin{aligned} SWF &= U_G + \Pi_B + \Pi_P - SC \\ &= \theta[B_0 + b(i) - \beta(e) + f(a)] - \theta n(a) - C_0 + c(i) + \gamma(e) - i - e - a \end{aligned} \tag{4.21}$$

The problem is solved as above:

$$\max_{i,e,a,d} SWF$$

with non-negativity constraints:

$$i, e, a, d \geq 0$$

The first order conditions are:

$$\begin{cases} [\theta b_i(i^*) + c_i(i^*) - 1]\, i^* = 0 \\ [-\theta\beta_e(e^*) + \gamma_e(e^*) - 1]\, e^* = 0 \\ [\theta f_a(a^*) - \theta n_a(a^*) - 1]\, a^* = 0 \end{cases}$$

where $h_j(j) = \dfrac{\partial h(j)}{\partial j}$ and j^* is the efficient level of investment j.

We are still keeping Hart's assumptions about the solutions for i and e. In addition, we assume that a still has an internal solution as well:

$$\begin{cases} \theta b_i(i^*) + c_i(i^*) = 1 \\ e^* = 0 \\ f_a(a^*) - n_a(a^*) = \frac{1}{\theta} \end{cases} \tag{4.22}$$

We are now ready to compare the relative inefficiencies of the three forms of provision.

Investments under private provision

Under private provision, the government signs two short-term contracts with two different private firms: B and P. The problem of the builder is therefore the following:

$$\max_{i,e} \Pi_B = \lambda_B - i - e$$

In the second period, the problem of the provider is:

$$\max_{a} \Pi_P = \lambda_P - OC$$

With the resulting investments:

$$i_p = e_p = a_p = 0$$

None of the firms has an incentive in investing: the builder considers i and e simply as a loss, as they do not influence its costs or productivity. The same holds for the provider and a.

Investments under partnership

When a partnership between B and P is formed, the consortium takes into account the effects of i and e on the provider's costs. The governments signs one long term contract with this consortium , whose problems is:

$$\max_{i,e,a} \Pi_{PPP} = \lambda_{PPP} - OC - i - e$$

With the resulting investments:

$$\begin{cases} c_i(i_{ppp}) = 1, \\ \gamma_e(e_{ppp}) = 1, \\ a_{ppp} = 0 \end{cases}$$

In this case the consortium takes into account the effect of i and e on the provider's costs. As regards i, this is still insufficient, as no weight is given to the social benefit. As for e, the lack of consideration for the its negative effect on social benefits leads to an overinvestment. The consortium still does not undertake any investment in a.

Investments under public provision

Finally, with public provision the government signs only one short term contract with the builder and then provides the service or good in-house. The problem of the builder is the same as with private provision:

$$\max_{i,e} \Pi_B = \lambda_B - i - e$$

In the second period, the public provider partially takes into account the effects of the investment a on the collective benefit (but also on its private one). The problem for the government is::

$$\max_a U_G = \theta CB + (1-\theta)PB$$

With the resulting investments:

$$\begin{cases} i_g = 0, \\ e_g = 0, \\ f_a(a_g) + (\frac{1-\theta}{\theta})n_a(a_g) = \frac{1}{\theta} \end{cases} \tag{4.23}$$

The builder does not make any difference about the identity of the provider, as long as it is not part of a consortium. Therefore no investment in i and e will be undertaken. The government is investing in a, as it correctly takes into account its effect on the social benefit. Nevertheless, this provision is still inefficient, as G does not consider the existence of social costs.

Comparison and comments

Hart's suggestions still hold but we now have to take into account a third option. Given our discussion, we can state that public provision is good when traditional privatization is better then partnership and social costs are low. Following Hart (2003), traditional private provision is better than partnership when the characteristics of the facility are easier to specify. When social costs are high, traditional private provision should be preferred. If the quality of the service is easier to measure, then the choice is between public provision and partnerships. With low social costs, public provision should be preferred. Proposition

14 formalizes this result.

Proposition 14 *When the government has a private benefit from the investment "a", the levels of investments provided under different provision choices are as follows:*

$$\begin{cases} i_p = i_g < i_{ppp} < i^* \\ e^* = e_p = e_g < e_{ppp} \\ a_{ppp} = a_p = 0 < a^* < a_g \end{cases} \tag{4.24}$$

Proof. The first two rows are part of Proposition 12 and have been proven above.

As regards a, from (4.22) we know that the efficient level of a, a^*, is positive: so it is always higher than $a_{ppp} = a_p = 0$.

Furthermore, from direct comparison of (4.22) and (4.23) and given our concavity assumptions about the functions, we have that:

$$a^* < a_g$$

as

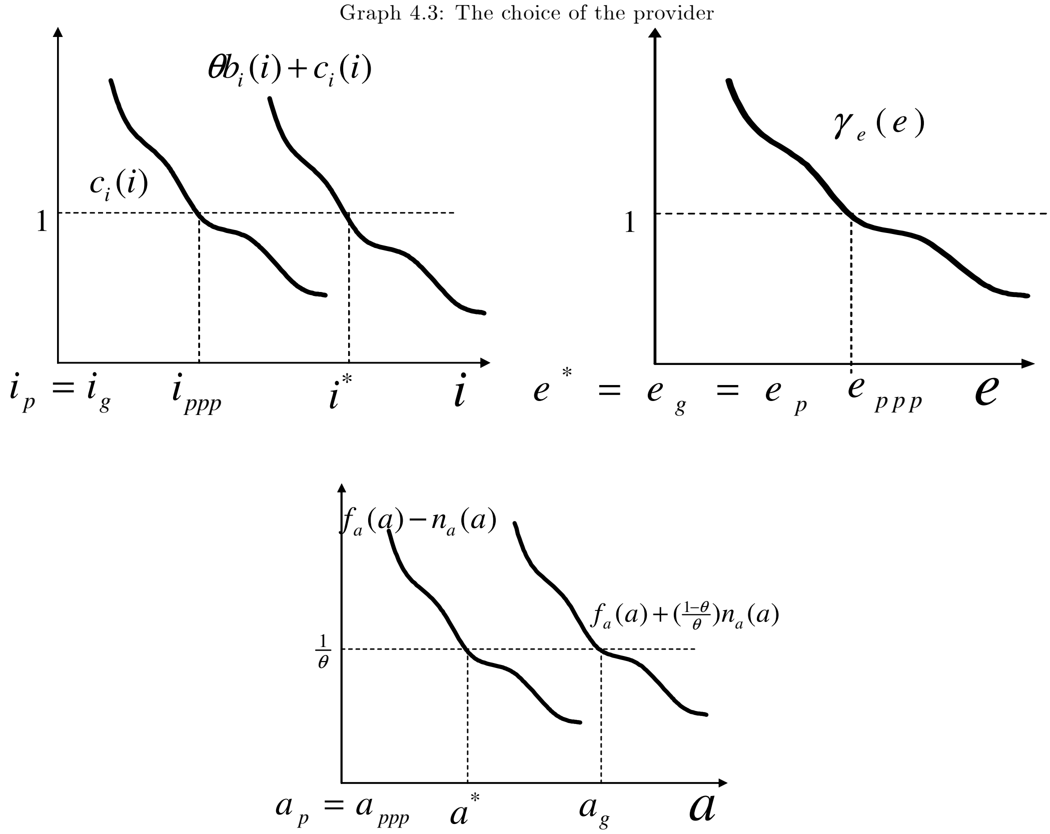
$$\frac{1-\theta}{\theta} > -1, \ \forall \theta \in (0,1)$$

Graph 4.3 shows the relations among i, e and a under different provision choices. The best choice is the one maximizing the SWF, as defined in (4.21). In table 4.4 we compare the values taken by the SWF under different provision schemes. The first best is never reached; from comparisons in (4.24) and in Table 4.4, and recalling Proposition 12, Proposition 14 directly follows.

Table 4.4: The optimal choice

Private provision:	SWF^p :	$\theta B_0 - C_0$
Public provision:	SWF^g :	$\theta[B_0 + f(a_g) - n(a_g)] - C_0 - a_g$
Partnership	SWF^{ppp} :	$\theta[B_0 + b(i_{ppp}) - \beta(e_{ppp})] - C_0 + c(i_{ppp}) + \gamma(e_{ppp})$
		$-i_{ppp} - e_{ppp}$

Graph 4.3: The choice of the provider



■

Public provision is unambiguously bad when the government can easily use the facility (or the service itself) to maximize its private benefit. That is, when the social cost is very relevant or even bigger than the social benefit (in which case, a should probably be equal to 0). For instance, in Italy, schools are typically seen as places where hiring teachers is a fast (but very costly) way of increasing consensus. The model suggests that education could be efficiently left to private providers to avoid overstaffing. Prisons, on the contrary, appear to have very low "electoral" characteristic (overstaffing, for instance, is never an issue), therefore public provision could be better.

The fact that, in reality, the provision of a lot of the collective goods is not (fully) privatized (e.g.: education) does not weaken our conclusions. In fact, our model provides a rationale for inefficient government's choices: if the private benefit associated to the provision of a good is big enough, then the government, who actually decides about the identity of the provider, does not want to lose it. We state this result more formally in Lemma 8.

Lemma 8 *The choice of the government is inefficiently biased towards public provision.*

Proof. We can compare the government's actual choice, based on U_G, evaluated for different provision choices, and the optimal one, based on SWF, evaluated in the same values. We summarize this problem in Table 4.5.

Table 4.5: The government's choice

	U_G
Private provision:	U_G^p : $SB - \lambda_B - \lambda_P = \theta B_0 - C_0$
Public provision:	U_G^g : $SB - \lambda_B - OC = \theta[B_0 + f(a_g)] + (1-\theta)n(a_g) - a_g$
Partnership	U_G^{ppp} : $SB - \lambda = \theta[B_0 + b(i_{ppp}) - \beta(e_{ppp})] - C_0 + c(i_{ppp})$
	$+\gamma(e_{ppp}) - i_{ppp} - e_{ppp}$

As $(1-\theta)n(a) > -\theta n(a)$ for any positive value of the social cost $n(a)$, the government's utility is distorted towards public provision, whereas the choice between partnership and traditional privatization is still (constrained) optimal. ■

It is worth noting that, in the analysis above, we could substitute the government with a not-for-profit institution, if we think that G is always incapable of providing the service in-house. In the case of a not-for-profit provider, our model recalls the one in Dixit (2002b). Beside the interest for the social benefit, these organizations might have also private benefits (e.g.: the spread of their religious beliefs) which are not necessarily compatible with public interests.

4.5 Conclusions

The aim of this chapter was to introduce new and more complete models of partnerships. In particular, we distinguish from the existing literature in our effort to introduce the government as an alternative provider and in studying the more efficient employment choices.

There is a trade off given by the choice of a public provider: the government G is the only employer which is able to provide a more efficient incentive to the worker. Nevertheless, if G sees the opportunity of exploiting these investments for a private benefit (for instance, for higher probability of election), then the scope for public provision is dramatically reduced. Public provision, as opposed to partnerships, is also particularly good when employer's and employee's effort are complementary and relevant. Our models seem to suggest that health service, usually requiring very expensive investments by the provider, should be kept under the public sector. On the contrary, education can be contract out to the private sector (in particular, to partnerships). This is especially true in countries where employing teachers is considered a way of buying consensus.

Our model also explains wage differentials in the private and the public sector: public workers might accept lower wages, due to the ability of the public provider to increase the worker's utility through the investment in a.

Finally, our model stresses that often the government choice is financially constrained. In this case, the solution is biased towards the private sector, and in particular partnerships, which are the cheapest alternative.

Further research should be aimed at developing at least the following problem: what would happen if there were different types of workers or different types of workers' efforts (i.e.: laziness)? We may expect some workers to be more efficiently hired and managed by a private provider, so that a further trade off emerges. The model above is indeed very incomplete and too counter-intuitive: it might further explain "public service motivation" as in Francois (2000) and the presence of higher wages in the public sector: they are paid to compensate the lower satisfaction of the workers. Nevertheless, in reality private workers are often seen as very productive whereas in our model they appear to be lazier.

4.6 Appendix: Proof of the condition in (4.13)

We now want to show that the condition:

$$\delta^{\beta}\left(\frac{1+\delta(1-\beta)}{1-\beta}\right)^{1-\beta} < 1+\delta$$

is always satisfied for any $\beta, \delta \in (0,1)$.

First of all, we apply a logarithmic transformation (which is positive monotonic) to both the sides of the inequality:

$$Log\left[\delta^{\beta}\left(\frac{1+\delta(1-\beta)}{1-\beta}\right)^{1-\beta}\right] < Log\left[1+\delta\right]$$

Therefore:

$$\beta Log\delta + (1-\beta)Log[1+\delta-\delta\beta] - (1-\beta)Log[1-\beta] - Log[1+\delta] < 0$$

We evaluate the function on the left hand side in $\beta = 0$ and obtain:

$$Log[1+\delta] - Log[1] - Log[1+\delta] = 0$$

As $\beta \in (0,1)$, it is now sufficient to show that the first derivative with respect to β of the function on the left hand side of the inequality is always negative:

$$\begin{gathered}\tfrac{\partial}{\partial\beta}\beta Log\delta + (1-\beta)Log[1+\delta-\delta\beta] - (1-\beta)Log[1-\beta] - Log[1+\delta] = \\ Log\delta - \delta\frac{1-\beta}{1+\delta-\delta\beta} - Log[1+\delta-\delta\beta] + \frac{1-\beta}{1-\beta} + Log[1-\beta] < 0\end{gathered}$$

which reduces to:

$$1 < Log[\frac{1+\delta-\delta\beta}{\delta(1-\beta)}]^{1+\delta-\delta\beta}$$

This condition is satisfied when:

$$(\frac{1+\delta-\delta\beta}{\delta(1-\beta)})^{1+\delta-\delta\beta} > e \tag{4.25}$$

where e is the Napier's constant[17]. It is easier to solve this inequality in (4.25) if we reduce it to a one variable problem; we set:

$$1 + \delta - \delta\beta = x$$

and it can be easily checked that, for $\beta \in (0, 1)$ and $\delta \in (0, 1)$, we have:

$$1 \leq x \leq 2$$

After the necessary transformations, (4.25) becomes:

$$[\frac{x}{x-1}]^x > e \tag{4.26}$$

The function $\frac{x}{x-1}$ is monotonically decreasing (its slope is always negative); so must be $(\frac{x}{x-1})^x$, as it is obtained as a positive monotonic transformation of the former function. The function $(\frac{x}{x-1})^x$ reaches its minimum when x equals its maximum possible value $(= 2)$. If (4.26) is satisfied for this $x = 2$, then it is *a fortiori* true for smaller values of x:

$$\left(\frac{2}{1}\right)^2 = 4 > e$$

Q.E.D.

[17]The constant e is occasionally called Euler's number after the Swiss mathematician Leonhard Euler, or Napier's constant in honour of the Scottish mathematician John Napier who introduced logarithms. In honour of the University where I studied and of the Country I've been living in for the last four years, I think Napier's constant is more appropriate.

Chapter 5

Conclusions

This is the closing and concluding chapter of the thesis. We do hope that the readers would agree with us that playing games has been a fascinating activity.

Nevertheless, as we explained in the Introduction, we were not here simply to play or win anything; we were here to learn something about the world. In particular, we wanted to explain, or solve, three of what we regard as main political puzzles nowadays:

1) Why is delegation so widespread and is it an efficient way of increasing power?

2) Is there any way we can manipulate voting in public bodies in order to guarantee optimal social choices?

3) What is the best provision mechanism for public (or collective) goods and services?

Yet, before answering these questions, we would like to draw the reader's attention to one final and unexpected common element of the three chapters, that is to say multiplicity of equilibria. This is an *ex-post* problem, which emerges in chapter two and three in particular, and that we could not forecast *a priori*. It was not our aim either to analyze multiplicity or to introduce refinements. The reason for this choice is that, rather than being interested in which particular equilibria our subjects were reaching, we preferred to focus on the general characteristics of these equilibria. For instance, the result that no first mover advantage exists in delegation games, and the result of a compensating voting behaviour in chapter three, are themselves worthy of comments. Moreover, it is also interesting to notice how this theoretical multiplicity can be dramatically reduced, if not even wiped out, just by introducing more realistic elements in our models. For example, just by introducing delegation costs in the bargaining process. What we obtained was a refinement through an addition of realism, rather than through excessive, and sometimes unconvincing, strong rationality.

We now leave this multiplicity *excursus* and turn to our principal question: can game theory help us to solve our political problems, or was it just for fun (and for sake of a title) that we wrote a thesis? Surely, game theory has shown great power in explaining the situations under analysis. Grasping the terms of a matter is a necessary first step towards its solution. In particular, we now better understand the source of inefficiency in our environments. Delegation, for instance, is a dominant strategy, and that is why it is played even if the final outcome may be lower than without it. Lack of coordination and the presence of self interested voters are the sources of inefficiency in the third chapter. In that case, the order of vote turns out to be an extremely powerful implementation mechanism, at least in small committees. Finally, inefficient levels of investments in chapter four are a consequence of lack of proper incentives for the agents.

But can we actually solve all these problems? Are our models of any help? Can we avoid inefficiencies or are we doomed simply to observe them? Our analysis offers some possible solutions.

1) The analysis in the second chapter is mainly positive rather than normative. Principals cannot be forced to coordinate, unless some public regulation is introduced. This is a desirable solution when efficiency losses from the prisoner's dilemma are particularly wide. For instance, in small firms direct negotiation should be promoted, when a delegated bargaining process would be excessively time consuming or the legal/training costs for the representative workers would be too high. Nonetheless, it seems extremely difficult to regulate delegation, especially if delegation costs are idiosyncratic.

2) Authorities taking decisions should have rules governing and regulating the voting system. When these authorities are organized as small committees and members might have different preferences, then an optimal order of vote can be found and imposed. The limit of this solution is that it does not seem politically feasible to describe some members of public authorities as biased or selfish. A wider range of action is probably feasible for voting rules in boards of directors or similar bodies.

3) There exist areas where any public authority (government or municipality) can *efficiently* run public services, though in a second best scenario. According to our model, this is especially true, for instance, in the field of health services. The debate about privatization, which tends to focus mainly on efficiency to sustain market solutions, should take into account our findings. Nevertheless, budget constraints and the possibility of using public investments for electoral purposes limit the scope for public provision.

Further ideas for future research

We want to add one final remark before concluding, which could provide further ideas for future research.

At a closer look, and admittedly beyond our original aim, our three political puzzles seem to depict quite a general and single scenario. Delegation is the heart of representative democracy: parliaments and governments are delegated bodies, who should act as agents for the society (the principal). So, is representative democracy an efficient way of organizing public life? Furthermore, we know that agents do not necessarily share the principals' objective: is there a device to minimize the bias of self interested agents? And finally, once we set up a set of rules for governing a society, what is the best provision mechanism in the context of public goods?

Answers are not easy and maybe the models we described are far too peculiar to address such general questions. For instance, in chapter two the principals are single subjects with clear preferences (maximizing - or minimizing - the selling price), whereas we know that aggregating social preferences could be a very demanding task. In addition, our main result in chapter three (that is, there exists a socially optimal order of voting) surely holds for small committees, but we are still not ready to generalize it to wider assemblies. Finally, we understand that the economy we describe in chapter four is far too simplified, with one single worker (or one single type of workers) representing the entire consumers/workers set (or, to be consistent with the present discussion, the electoral body).

Answers are not easy, we stated. But nonetheless answers must be there, somewhere. Finding them is a task we here personally undertake and extend to anyone who, like us, believe in the explaining power of economic theory.

And in the richness of a world still far from being completely understood.

And finally, of course, in the fascination of games.

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